



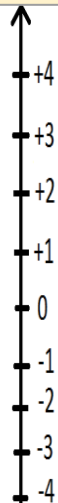
# Knowledge Organisers

## Year 7

## CORE

- Recall times tables accurately e.g  $7 \times 8 = 56$  or  $9 \times 9 = 81$
- For order of operations we use BIDMAS
  - Brackets
  - Indices
  - D/M divide and multiply
  - A/S add and subtract
- A negative number is less than zero
  - Negative ( $\times/+$ ) negative gives a positive answer
  - Negative ( $\times/+$ ) positive gives a negative answer
  - ( $+$ ) means subtract - ( $-$ ) means add
- Understand place value - Place value is the value of each digit in a number. For example, the 5 in 350 represents 5 tens, or 50; however, the 5 in 5,006 represents 5 thousands, or 5,000
 

T	H	H	T	U
5	0	0	6	
- Rounding - if 5 or more round up, if less than 5 stays the same
- There are 60 seconds in a minute, there are 60 minutes in an hour, there are 24 hours in a day
- $\frac{1}{4}$  hr = 15 mins,  $\frac{1}{2}$  hr = 30 mins  $\frac{3}{4}$  hr = 45 mins
- There are 100 pence in a pound, money is always rounded to 2d.p.



### Keywords:

- Integer - A whole number ie. -1 or 4
- Square number - The result of multiplying an integer by itself ie.  $3 \times 3 = 9$
- Factor - a number that divides a number without a remainder ie. 5 is a factor of 10.
- Multiple - The times tables of a number ie. 8 is a multiple of 2
- Prime number - A number that has only two factors, 1 and itself ie. 11 is a prime number.
- Decimal place - The amount of numbers after a decimal point. It is normally written as d.p.
- Significant figure - The significant digits of a number are the digits that have meaning or contribute to the value. We start counting significant figures from the first non-zero figure. I.e. 0.086, the 8 is the first significant figure. It may be written as s.f.

## GOOD TO KNOW...

- Cube numbers - A number multiplied by itself, then multiplied by itself again ie.  $2 \times 2 \times 2 = 8$
- Square root - Is the inverse of a square number ie. the square root of 16 is 4 as  $4 \times 4 = 16$
- Cube root - Is the inverse of a cube number ie. the cube root of 27 is 3 as  $3 \times 3 \times 3 = 27$
- Alternate meanings
  - Multiply is the same as times
  - Subtract is the same as take away
  - Product means multiply
  - Sum means add
- Inverse operations mean do the opposite
  - The inverse of add is subtract
  - The inverse of multiply is divide
- 1 is not a prime number as it only has one factor. 2 is the only even prime number.
- HCF - highest common factor
- To find the HCF of two or more numbers:
  - List all factors of both numbers
  - Find the highest factor that appears in both lists
- LCM - lowest common multiple
- To find the LCM of two or more numbers:
  - List multiples of both numbers
  - Repeat until you find the lowest common multiple
- Estimate - Rounding of a number to make a calculation easier ie. we can estimate  $99.6 \div 7.2$  to be  $100 \div 10 \approx 10$
- Round 4953 to 2 s.f.

## HOW TO....

$$57 \div 3 = 19$$

How many times does 3 go into 5?

It goes into 5 once and has a remainder of 2.

19

3 | 57

30

---

27

6

How many times does 3 go into 27?

It goes into 27 nine times and has no remainder.

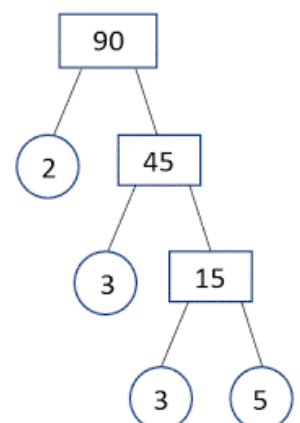
	2	4	X
0	6	1	3
1	2	2	6
8	6	4	

Blue arrow pointing from the multiplication table to the subtraction below.

	1		
	2		
	6	3	
	4	7	
	2	5	2
	0	6	1
	2	9	6
	1	8	3

← Answer

	3	1
3	4	3
-	2	3
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1	0	6

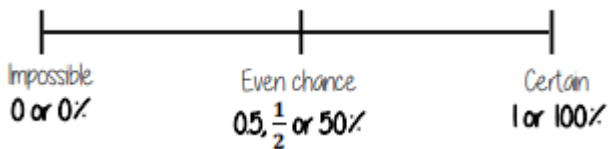


Stay or round up?

4	9	5	3
<hr style="border: 1px solid blue;"/>			
5	0	0	0

CHECK

## CORE



Probability can be a fraction, decimal or percentage.

Probability is **always** a value between 0 and 1

### Keywords

**Set:** Collection of things

**Element:** Each item in a set is called an element

**Mutually exclusive:** Events that do not occur at the same time

**Probability:** Likelihood of an event happening

**Bias:** a built-in error that makes all values wrong (unequal) by a certain amount, eg a weighted dice

**Fair:** There is zero bias, and all outcomes have an equal likelihood

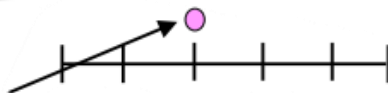
**Random:** something happens by chance and is unable to be predicted.

## GOOD TO KNOW...

The more likely the event the further up the probability it will be in comparison to another event



There are 2 pink and 2 yellow balls, so they have the same probability.



There are 5 possible outcomes, so 5 intervals on this scale, each interval value is  $\frac{1}{5}$

The **universal set** has this symbol  $\xi$  - this means **EVERYTHING** in the set.

$\xi = \{\text{the numbers between 1 and 50 inclusive}\}$

$$\frac{4}{10} = \frac{40}{100} = 0.40 = 40\%$$

## HOW TO....

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$$



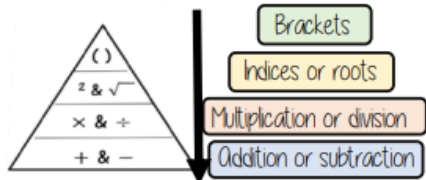
Probability =  $\frac{\text{number of times events happen}}{\text{total number of possible}}$

$P(\text{Blue}) = \frac{4}{10}$  ← There are 4 blue sectors

← There are 10 sectors overall

Probability notation P(event)

## CORE



### Keywords

**Expression:** a maths sentence with a minimum of two numbers and at least one Maths operation (no equals sign)

**Linear:** the difference between two terms increases or decreases by the same value each time

**Output:** the number / expression that comes out of a function

**Formula:** a fact or rule that is expressed in terms of mathematical symbols

**Function:** a relationship that instructs how to get from an input to an output

**Integer:** A whole number that is positive or negative

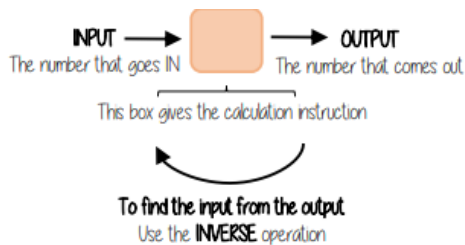
**Factorise:** To factorise an expression fully, means to put it in brackets by taking out the highest common factors

**Inverse:** the operation that undoes what was done by the previous operation (the opposite operation)

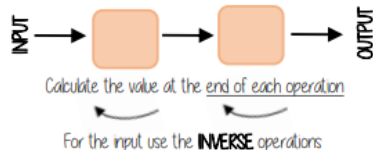
**Equation:** a formula that expresses the equality of 2 expressions by connecting them with the equals sign =

## GOOD TO KNOW...

$5 + 5 + 5$	$y + y + y + y$	$20 - h$
$3 \times 5$	$y \times 4$	$\frac{20}{h}$
$5 \times 3$	$4 \times y$	$\uparrow$
	$4y$	

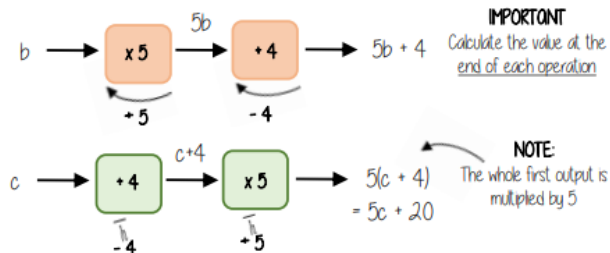


### Two step function machines

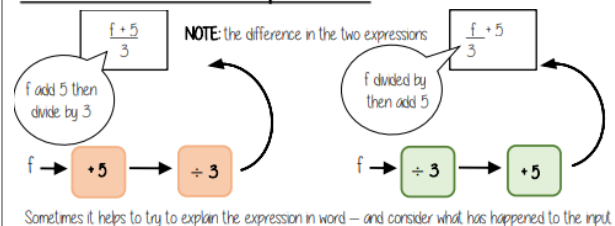


## HOW TO....

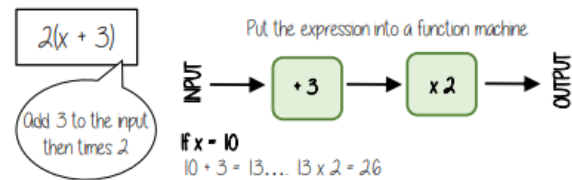
### Two step function machines (algebra)



### Find functions from expressions



### Substitution into an expression





## CORE

## GOOD TO KNOW...

## HOW TO....

### Keywords

**Mean:** a calculated 'central' value of a set of data

**Mode:** The number which appears most often in a set of numbers.

**Median:** the median is the middle value in a numerically sorted set of numbers

**Range:** the difference between the highest and lowest values.

**Tally:** a simple way of recording and counting frequencies

**Modal class:** the class interval with the highest frequency

**Frequency:** how often something occurs

**Discrete data:** Data that can only take certain data  
E.g. Number of students, shoe sizes, number of test questions answered.

**Continuous data:** data that can be measured on an infinite scale, It can take any value between two numbers, no matter how small.

E.g. Height, weight, temperature, length

A **pictogram** is a chart or graph which has pictures to represent data

A **bar graph** is a graph that is made up of bars with different heights. Each bar represents a different category. The height of each bar shows the number of items, or how often something happens.

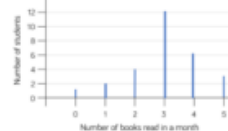
### Pictogram

Language	
French	●●●●●
Spanish	●●●●●
German	●

● = 4 people

- Need to remember a key
- Visually able to identify mode

### Line Chart



- Gaps between the lines
- Clearly labelled axes
- Scale for the axes
- Discrete Data

	Data Title	Tally	Frequency
Grouped or ungrouped categories			

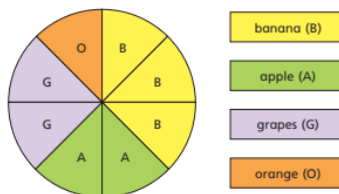
Total number of that group observed

### Bar Chart



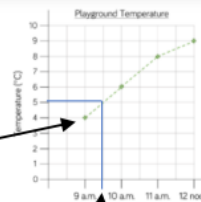
- Gaps between the bars
- Clearly labelled axes
- Scale for the axes
- Title for the bar chart
- Discrete Data

The pie chart shows the favourite fruit of 48 children.



### Draw and interpret line graphs

- Commonly used to show changing over time
- The points are the recorded information and the lines join the points



Line graphs do not need to start from 0

More than one piece of data can be plotted on the same graph to compare data

It is possible to make estimates from the line  
e.g. temperature at 9.30am is 5°C

### Draw and interpret Pie Charts

Remember a circle has 360°

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

There were 60 people asked in this survey  
(Total frequency)

$\frac{32}{60}$  "32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs



**Multiple method**  
As 60 goes into 360 - 6 times  
Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

$$\frac{32}{60} \times 360 = 192^\circ$$

Use a protractor to draw  
This is 192°

Represents quantitative, discrete data

Hey diddle diddle the median's the middle  
You add and divide to find the mean,  
The mode is the one that appears the most,  
And the range is the difference between them.

## CORE

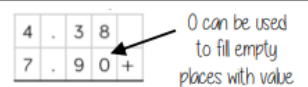
## GOOD TO KNOW...

## HOW TO....

### Common Metric Units

<b>Length</b>	cm centimetre m metre km kilometre
<b>Area</b>	cm <sup>2</sup> square centimetre m <sup>2</sup> square metre km <sup>2</sup> square kilometre
<b>Volume</b>	cm <sup>3</sup> cubic centimetre m <sup>3</sup> cubic metre
<b>Mass</b>	g gram kg kilogram t tonne
<b>Capacity</b>	ml millilitre l litre
<b>Time</b>	s second min minute h hour
<b>Temperature</b>	°C degrees celsius

### Addition/ Subtraction with decimals



The decimal place acts as the placeholder and aligns the other values

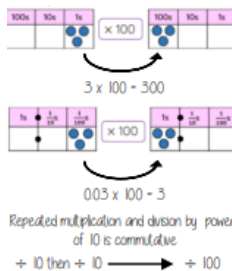
$$\begin{array}{r}
 4 \quad 1 \\
 5.2 \\
 - 3.6 \\
 \hline
 1.6
 \end{array}$$

Order 4.944, 5.224, 5.74, 5.455 from greatest to least.

largest smallest  
 4.944, 5.224, 5.74, 5.455  
 least smallest largest between

5.74, 5.455, 5.224, 4.944

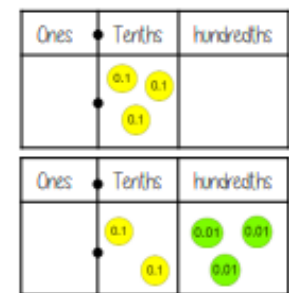
### Multiply/ Divide by powers of 10



Repeated multiplication and division by 10 is commulative  
 + 10 then + 10 → + 100

### Comparing decimals

Which the largest of 0.3 and 0.23?



0.3 > 0.23  
 "There are more counters in the furthest column to the left"

0.30 }  
 0.23 }  
 Comparing the values both with the same number of decimal places is another way to compare the number of tenths and hundredths

- Rounding - if 5 or more round up, if less than 5 stays the same e.g. 5.6 to the nearest whole number is 6.
- When multiplying and dividing decimals by powers of ten, set up your decimal in a place value table and move the digits (to the left if multiplying or to the right if dividing) by the amount of zeros in the power of ten e.g.  $3.2 \times 10$  means we move each digit one space to the left so the answer is 32.

**Keywords**

**Rounding:** making a number simpler but keeping its value close to what it was

**Estimate:** a way of approximately calculating an answer

**Approximate:** A result that is not exact, but close enough to be used

**Multiples:** the product result of one number multiplied by another number

**Order of Magnitude:** If one amount is an order of magnitude larger than another, it is ten times larger than the other

**Decimal:** a number with a decimal point used to separate ones, tenths, hundredths etc

**Length:** how far from end to end

**Mass:** mass is often called weight but mass and weight are not the same

**Capacity:** the amount a container or something can hold

## CORE

**Face:** the flat surface of a solid object

**Edge:** the line that joins corners or surfaces of a shape

**Vertice:** the points where two or more line segments or edges meet (like a corner)

**Plan:** a scale drawing showing a 3D shape when it is looked at from above

**Elevation:** the view of a 3D shape when it is looked at from the side or from the front

**Perimeter:** The distance around a two-dimensional shape. To calculate the perimeter, add the length of all sides of the shape.

**Area:** the total space taken up by a flat (2-D) surface or shape of an object.

**Regular:** a regular shape has all sides equal and all angles equal

**Irregular:** an irregular shape has at least one side different to the other sides, or angle different to the other angles

**Surface area:** the total area of all of the faces

**Polygon:** a flat two-dimensional shape with straight sides that are fully closed

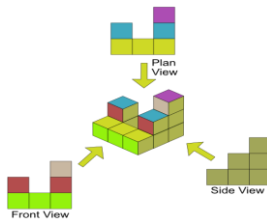
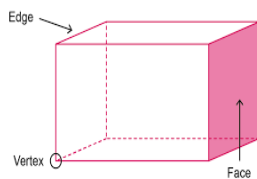
**Compound or composite shape:** any shape that is made up of two or more geometric shapes

**Parallel:** Lines on a plane that never meet

**Parallelogram:** a quadrilateral with opposite sides parallel and equal

## GOOD TO KNOW...

Name of 3-D shape:	Picture of 3-D shape:	Attributes:
Cube		Faces : 6 Edges : 12 Vertices : 8
Rectangular Prism or Cuboid		Faces : 6 Edges : 12 Vertices : 8
Sphere		Curved Face : 1 Edges : 0 Vertices : 0
Cone		Flat Face : 1, Curved Face : 1 Edges : 1 Vertices : 1
Cylinder		Flat Face : 2, Curved Face : 1 Edge : 2 Vertices : 0



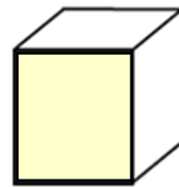
$P = a + b + c + d$   
 $P = 17 + 8 + 17 + 8$   
 $P = 50 \text{ ft}$

$P = 2 \times (\text{length} + \text{width})$   
 $P = 2 \times (17 + 8)$   
 $P = 2 (25)$   
 $P = 50 \text{ ft}$

Shape	Name	Formula for Area
	Square	Base x Height
	Rectangle	Base x Height
	Triangle	Base x Perpendicular Height + 2

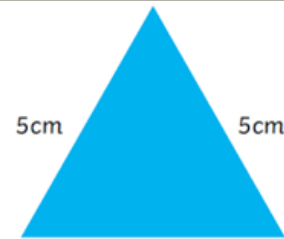
## HOW TO....

What is the volume of the cube?



2 cm

$$2 \text{ cm} \times 2 \text{ cm} \times 2 \text{ cm} = 8 \text{ cm}^3$$



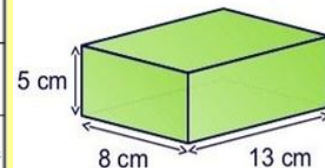
$$5 + 5 + 5 = 15 \text{ cm}$$

Perimeter = 15 cm

## Area of squares and rectangles

Area = 5 x 2 metres  
 Area = 10m<sup>2</sup>

Area = 4 x 4 cm  
 Area = 16cm<sup>2</sup>



Volume of cuboid

$$= \text{length} \times \text{width} \times \text{height}$$

$$= 5 \times 8 \times 13$$

$$= 520 \text{ cm}^3$$

## CORE

## GOOD TO KNOW...

## HOW TO....

**Integer:** A whole number that is either positive or negative. E.g. 3, 100, -12 are all integers.

**Significant figure:** A digit that gives meaning to a number. The most significant digit (figure) in an integer is the number on the left. The most significant digit in a decimal fraction is the first non-zero number after the decimal point. E.g. 320 rounded to 1 s.f. would be 300.

**Fraction:** how many parts of a whole we have. E.g.  $\frac{1}{2}$  is a fraction.

**Place value:** the numerical value that a digit has decided by its position in the number.

**Placeholder:** a number that occupies a position to give value

**Numerator:** the number above the line on a fraction. The top number. Represents how many parts are taken

**Denominator:** the number below the line on a fraction. The number represent the total number of parts..

**Whole:** a positive number including zero without any decimal or fractional parts

**Unit fraction:** a fraction where the numerator is one and denominator a positive integer.

**Dividend:** the amount you want to divide up

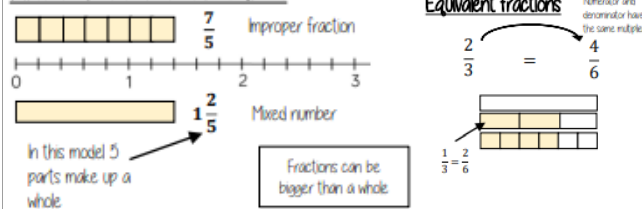
**Divisor:** the number that divides another number

**Quotient:** the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient

**Factors:** integers that multiply together to get the original value. E.g. 3 and 4 are both factors of 12.

**Scale factor:** the multiple that increases/ decreases a shape in size

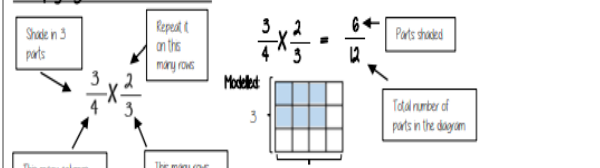
### Mixed numbers and fractions



### Add/Subtraction any fractions



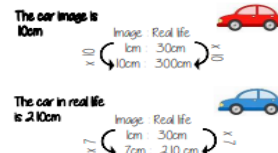
### Multiplying non-unit fractions



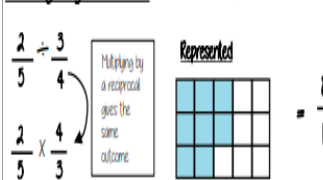
### Draw and interpret scale diagrams

A picture of a car is drawn with a scale of 1:30

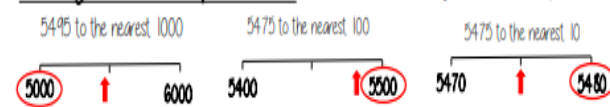
For every 1cm on my image is 30cm in real life



### Dividing any fractions



### Rounding to the nearest power of ten



### Round to 1 significant figure

370 to 1 significant figure is 400

37 to 1 significant figure is 40

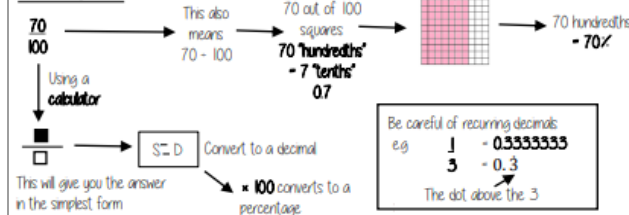
37 to 1 significant figure is 4

0.37 to 1 significant figure is 0.4

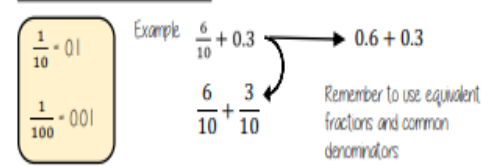
0.00000037 to 1 significant figure is 0.0000004

Round to the first non zero number

### Convert FDP



### Fractions and decimals



## CORE

**Percent:** parts per 100 – written using the % symbol

**Decimal:** a number in our base 10 number system. Numbers to the right of the decimal place are called decimals

**Equivalent:** of equal value

**Term:** a single number or variable. E.g, 3, x, 5x are all terms.

**Index form:** A system of writing very big or very small numbers

$$5 \times 5 \times 5 \times 5 = 5^4$$

$$a \times a \times a \times a \times a \times a = a^6$$

**Negative indice:** A power (indice) below zero

**Standard form:** A system of writing very big or very small numbers

300 000 can be written as  $3 \times 10^5$

0.035 can be written as  $3.5 \times 10^{-2}$

**Power:** The exponent – or the number that tells you how many times to use the number in multiplication

E.g. the number  $4^5$  shows that 5 is the power

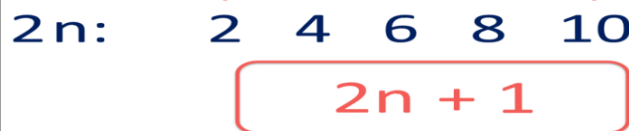
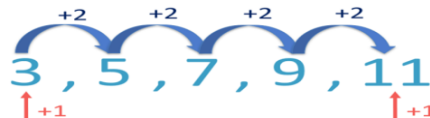
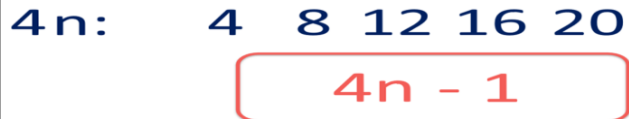
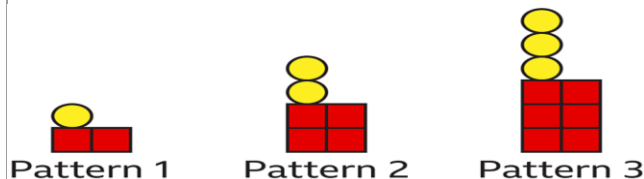
**Exponent:** The power – or the number that tells you how many times to use the number in multiplication

**Surface area:** Area of the faces of a 3d shape. Surface area is measured in units<sup>2</sup>

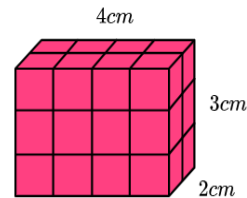
**Volume:** is the amount of space a 3D shape takes up. Volume is measured in units<sup>3</sup>.

## GOOD TO KNOW...

*Sequence:*



## HOW TO....



This cuboid is made from 24 unit cubes.

Its volume is

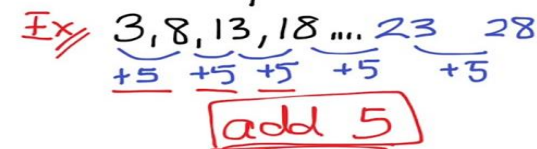
$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

$$\text{Volume} = 2 \times 4 \times 3$$

$$\text{Volume} = 24 \text{ cm}^3$$

Describing number sequences term-to-term rule

A sequence is a set of numbers described by a rule.



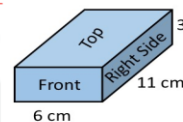
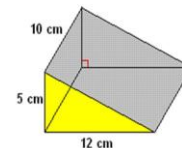
$$V = \frac{(b \times h) \times H}{2}$$

$$V = \frac{(12 \times 5) \times 10}{2}$$

$$V = \frac{60 \times 10}{2}$$

$$V = 30 \times 10$$

$$V = 300 \text{ cm}^3$$



Front		3 = 18	Front & Back (2 × 18) = 36
Right Side		3 = 33	Right & Left Side (2 × 33) = 66
Top		6 = 66	Top & Bottom (2 × 66) = 132

Total Surface Area = 234 cm<sup>2</sup>



## CORE

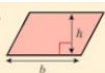
## GOOD TO KNOW...

## HOW TO....

### A parallelogram has:

- Two pairs of opposite sides that are equal in length.
- Two pairs of parallel sides
- Two pairs of equal opposite angles
- Diagonals** that **bisect** each other.
- No reflection symmetry
- Rotational symmetry of order 2**

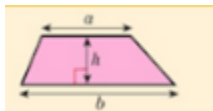
area of parallelogram = base length  $\times$  perpendicular height,  
 $A = bh$



### A trapezium has:

- One pair of unequal parallel sides.
- Diagonals that are not equal in length.
- No **reflection symmetry**
- No rotational symmetry.

$$\text{Area Trapezium} = \frac{a+b}{2} \times h$$



Where  $a$  and  $b$  are the parallel sides

Where  $h$  is equal to the perpendicular height of the trapezium

The **area of compound shapes** (also known as the area of composite shapes) is the amount of space inside a shape composed of basic shapes put together. It is measured in units squared ( $cm^2$ ,  $m^2$ ,  $mm^2$  etc.).

Compound shapes can also be called composite shapes.

To find the area of compound shapes we must divide the compound shape into basic shapes and find the area of each of the basic shapes and add them together.



Area of Shape A + Area of Shape B = Area of Compound Shape

### Common misconceptions

- Using incorrect units for the answer**

A common error is to forget to include squared units when asked to calculate area.

- Forgetting to convert measures to a common unit**

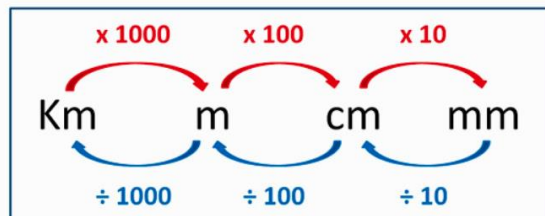
Before calculating the area of a trapezium, pupils must look at the units given in the question. If different units are given e.g. length =  $4m$  and width =  $3cm$  pupils must convert them either both to  $cm$  or both to  $m$ .

- Using length of the non-parallel sides when calculating area and not the height**

Sometimes in a question we are given additional lengths which are not needed in our calculations. Sometimes we are given the length of the non-parallel sides of the trapezium. We must be careful to not mistake these measurements for height.

- Length of the slanted side and perpendicular height**

Sometimes in a question we are given additional lengths which are not needed in our calculations. An example of this is being given the length of one of the slanted sides of the parallelogram. To calculate the area we must ignore the slant length and instead use the vertical height.



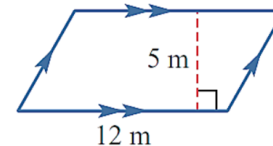
Find the area of the parallelogram.

$$A = bh$$

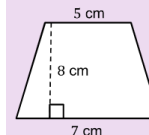
Substitute given values into the formula.

$$A = 5 \times 12$$

$$A = 60m^2$$



Calculate the area of the trapezium.

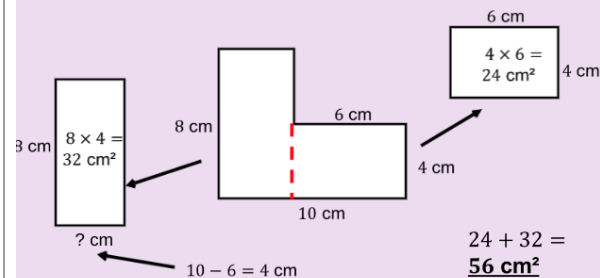


$$\text{Area of trapezium} = \frac{1}{2}(a + b)h$$

$$= \frac{1}{2} \times (5 + 7) \times 8$$

$$= 48cm^2$$

Calculate the **area** of this compound shape which is made up of two rectangles.



$$24 + 32 = \underline{56cm^2}$$

## CORE

## GOOD TO KNOW...

## HOW TO....

### Key Words

**Ratio and proportion** is an area of mathematics which deals with the relationship between two or more quantities.

**Ratio:** used to compare two or more numbers

**Highest Common Factor:** Is the greatest number that can be divided into two or more numbers without a remainder

**A ratio in its simplest form:** has been divided by the highest common factor of all the numbers in the ratio

**Equivalent ratios:** two ratios are considered equivalent if one can be expressed as a multiple of the other

**Dividing ratios** is a way of sharing a quantity in given parts of a ratio.

**Ratio to fraction:** When we express a ratio as a fraction, we need to know either the value of each part of the ratio, the sum of these will be or denominator

**Proportion** is a type of relationship between two variables

**Direct proportion** When two quantities are in direct proportion, as one increases the other does too.

**Unitary method:** uses a ratio in the form 1:n

### Common misconceptions

- Ratio written in the wrong order

A common error is to write the parts of the ratio in the wrong order.

E.g.

The number of dogs to cats is given as the ratio 12 : 13 but the solution is incorrectly written as 13 : 12.

- Ratios and fractions confusion

E.g.


The ratio 2 : 3 is incorrectly expressed as the fraction  $\frac{2}{3}$  and rather than the correct answer of  $\frac{2}{5}$ .

This is a misunderstanding of the sum of the parts of the ratio. The sum of all of the parts of the ratio gives us the denominator of the fraction.


The **unitary method** can be used to find the best value.

Which bag of apples is the best value?

1.2kg for  
£3.89



700g  
for  
£2.14



What is the price per gram?

$$\begin{array}{r} 1200g = 389p \\ + 1200 \quad + 1200 \\ \hline 1g = 0.324p \end{array}$$

$$\begin{array}{r} 700g = 214p \\ + 700 \quad + 700 \\ \hline 1g = 0.305p \end{array}$$

This is less money per gram, so it is the best value

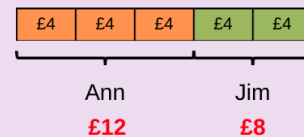
Simplify:

$$\begin{array}{c} 16 : 20 \\ \swarrow \quad \searrow \\ \div 4 \quad \div 4 \\ \hline 4 : 5 \end{array}$$

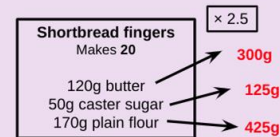
£20 is divided between Ann and Jim in the ratio 3 : 2

How much money does each get?

$$3 + 2 = 5 \text{ parts}$$



$$£20 \div 5 = £4$$

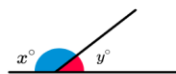


How much of each ingredient will be needed to make 50 fingers?

Hint → 20 : 50

## CORE

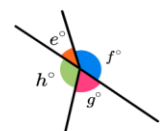
### Angles on a straight line



$$x + y + z = 180^\circ$$

(The sum of angles on a straight line equals  $180^\circ$ )

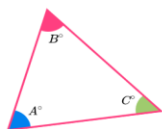
### Angles around a point



$$e + f + g + h = 360^\circ$$

(The sum of angles around a point equals  $360^\circ$ )

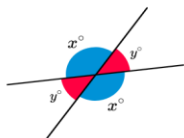
### Angles in a triangle



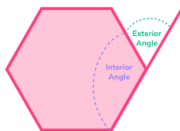
$$A + B + C = 180^\circ$$

(The sum of angles in a triangle equals  $180^\circ$ )

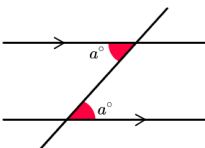
### Vertically Opposite angles



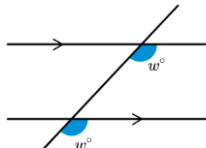
(Vertically opposite angles are the same size)



### Alternate angles are equal



### Corresponding angles are equal



## GOOD TO KNOW...



**Right angled triangle**  
One right angle  
 $90 + 55 + 35 = 180^\circ$

**Isosceles triangle**  
Two equal sides & angles  
 $72 + 72 + 36 = 180^\circ$

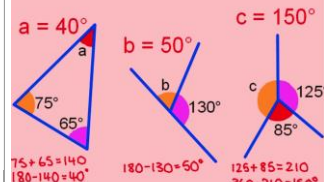
**Equilateral triangle**  
Three equal sides & angles  
 $60 + 60 + 60 = 180^\circ$

**Scalene triangle**  
All sides & angles different  
 $83 + 68 + 29 = 180^\circ$

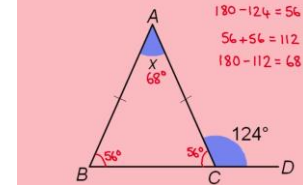
Quadrilateral	Image	Sides	Angles
Square		<ul style="list-style-type: none"> <li>Four equal sides</li> <li>Opposing sides are parallel</li> </ul>	<ul style="list-style-type: none"> <li>All equal angles (<math>90^\circ</math>)</li> </ul>
Rectangle		<ul style="list-style-type: none"> <li>Two pairs of equal sides</li> <li>Opposing sides are parallel</li> </ul>	<ul style="list-style-type: none"> <li>All equal angles (<math>90^\circ</math>)</li> </ul>
Parallelogram		<ul style="list-style-type: none"> <li>Two pairs of equal sides</li> <li>Opposing sides are parallel</li> </ul>	<ul style="list-style-type: none"> <li>Two opposing pairs of equal angles</li> </ul>
Rhombus		<ul style="list-style-type: none"> <li>Four equal sides</li> <li>Opposing sides are parallel</li> </ul>	<ul style="list-style-type: none"> <li>Two opposing pairs of equal angles</li> </ul>
Kite		<ul style="list-style-type: none"> <li>Two pairs of equal sides</li> </ul>	<ul style="list-style-type: none"> <li>One opposing pair of equal angles</li> </ul>
Trapezium		<ul style="list-style-type: none"> <li>One pair of parallel sides</li> </ul>	<ul style="list-style-type: none"> <li>No equal angles</li> </ul>
Isosceles Trapezium		<ul style="list-style-type: none"> <li>One pair of equal sides</li> <li>One pair of parallel sides</li> </ul>	<ul style="list-style-type: none"> <li>Two pairs of equal angles</li> </ul>
Irregular		<ul style="list-style-type: none"> <li>No equal side lengths</li> <li>No parallel sides</li> </ul>	<ul style="list-style-type: none"> <li>No equal angles</li> </ul>

## HOW TO....

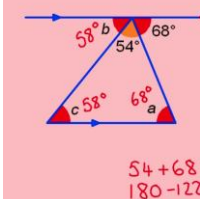
### Work out the size of angles a, b and c



### Calculate the size of the angle marked x.



### Calculate the size of angles a, b and c



### PARALLELOGRAM



opposite sides equal . . . . . Yes . .  
 opposite sides parallel . . . . . Yes . .  
 opposite angles equal . . . . . Yes . .  
 diagonals equal . . . . . No . .  
 diagonals cross at right angles . . . . . No . .  
 number of lines of symmetry . . . . . 0 . .  
 order of rotational symmetry . . . . . 2 . .

In order to solve problems using angle rules:

- 1 Identify which angle you need to find
- 2 Identify which angle rule/s apply to the context and write them down
- 3 Solve the problem using the angle rule/s. Give reasons where applicable
- 4 Clearly state the answer using angle terminology.



## CORE

## GOOD TO KNOW...

## HOW TO....

### Key Words

**Numerical Data:** Data that only takes numbers

**Stem and leaf diagrams** organise numerical data based on the place value of its numbers. Each number is split into two parts.

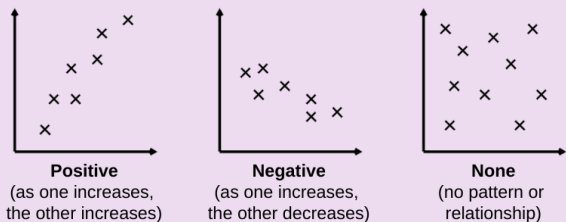
The first digit(s) form the **stem**

The last digit forms the **leaf**

**Scatter graphs** show the relationship between two sets of data, or two variables.

The relationship is described using **correlation**.

There are three main types of correlation:

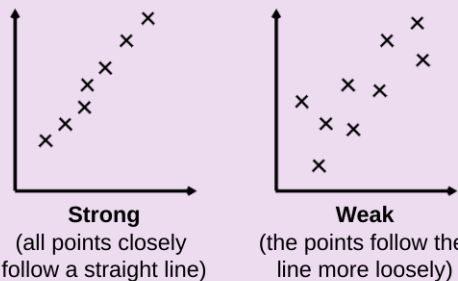


**Line of best fit** a straight line that goes through the middle of your points used to make estimates of other results..

A 'draw an ordered stem and leaf diagram' question is usually worth 3 marks:

- 1) All the data is ordered
- 2) You haven't missed any values
- 3) Include a key

Correlation can also be **weak** or **strong**.



### Line of best fit



### Draw a stem and leaf diagram:

- 1 Order the numbers from smallest to largest.

35, 50, 37, 44, 53, 41, 39, 45, 48, 56

becomes

35, 37, 39, 41, 44, 45, 48, 50, 53, 56

- 2 Split the numbers into two parts, the last part must be one digit only.

The number in our data will split into tens and units so 35 will be 3 and 5 (3 represents 30 and 5 is 5 units)

- 3 Put the values into the diagram and create a key.

Key : 3	5 represents 35
3	5 7 9
4	1 4 5 8
5	0 3 6

Represents the number 56

### Use a scatter diagram to make estimations

Temperature (°C)	21	26	15	23	18	29	20	27	22	17	30	19
Number of ice creams sold	70	86	50	80	58	96	66	92	74	54	100	62



## CORE

## GOOD TO KNOW...

## HOW TO....

### Key Words

**Coordinates** are locations of points on a grid

**Midpoint** - is the halfway point between two other coordinates

**2D shapes** are flat shapes which only have two dimensions; length and width

**A line of symmetry** shows where one side of a shape is the reflection of another

**The order of rotational symmetry** of a shape is how many times the shape fits onto itself during a  $360^\circ$  turn

**Transformations** change the size and/or the position of a shape. There are 4 transformations:

**Reflection** flips an object. The size and shape stay exactly but the shape is mirrored.

**Rotation** turns an object. The size and shape stay exactly the same but the orientation changes

**Translation** moves an object, the shape stays the same.

**Enlargement** changes the size of an object

**Scale factor** Indicates how many times bigger or smaller one shape is than another.

**Congruent** shapes are shapes that are exactly the same shape and size.

**Similar shapes** are the same shape, but different sizes.

**Object** The original shape before a transformation has

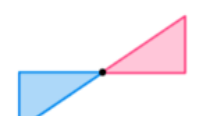
The **x-axis** and **y-axis** meet at the **origin**, (0, 0) where,

- the **x-axis** (the **horizontal axis**) is positive to the right of the origin, and negative to the left of the origin;
- the **y-axis** (the **vertical axis**) is positive above the origin, and negative below the origin.

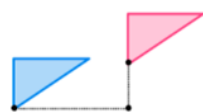
### 1. Reflection e.g.



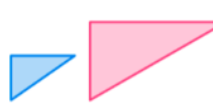
### 2. Rotations e.g.



### 3. Translation e.g.

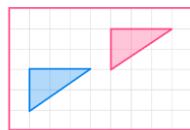


### 4. Enlargement e.g.



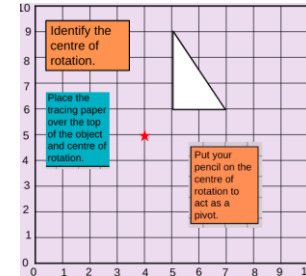
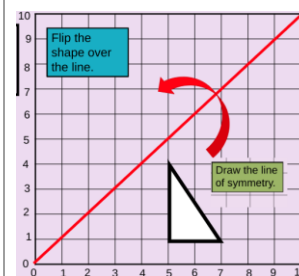
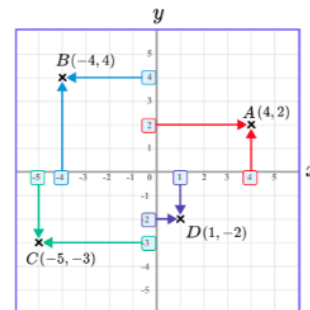
**Reflections, Rotations, and Translations** produce images that are **congruent**

These two triangles are congruent.



These two triangles are similar.

**Enlargements** produce images that are similar



We can use **column vectors** to describe translations.

For example:

$$\begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

This is the **x** value which tells us the left or right movement.

This is the **y** value which tells us the up or down movement.



# Knowledge Organisers

## Year 8

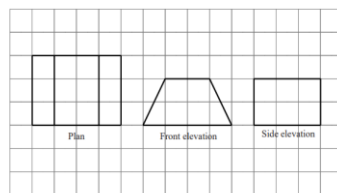
## CORE

- 2D - a 2 dimension shape (flat, e.g. square, circle)
- 3D - a 3 dimensional shape (solid, e.g. cube, cylinder)
- Volume - the space inside a 3D shape measured in cubed units
- Face - the flat surface 2D shapes which form a 3D solid
- Vertex/vertices - the corners of a 3D shape
- Edge - the lengths connecting the corners of a 3D shape
- Area - the space inside a 2D shape measured in squared units
- Area of a rectangle/square = length x width
- Area of a triangle =  $\frac{\text{base} \times \text{height (perpendicular)}}{2}$
- Volume of a cube/cuboid = length x width x height
- Volume of a prism = area of cross section (front face) x length
- Square has 4 equal sides
- Rectangle has 2 pairs of equal sides
- Triangle has 3 sides
  - An equilateral triangle has three equal sides and three equal angles of  $60^\circ$
  - An isosceles triangle has two equal sides where the two angles at the base of the triangle are equal
- Prism - a 3D shape which has the same cross section throughout, (e.g. cuboid, cylinder, triangular prism)

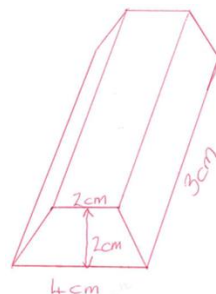
## GOOD TO KNOW...

- Plan is the view of a 3D shape when looked at from above
- Elevation is the view of a 3D shape when looked at from the front or the side

The diagram shows the plan, front elevation and side elevation of a solid shape, drawn on a centimetre grid.



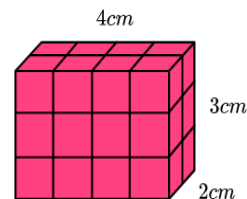
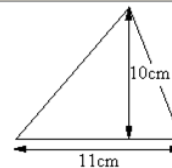
In the space below, draw a sketch of the solid shape. Give the dimensions of the solid on your sketch.



- Area of a trapezium =  $\frac{1}{2} \times (a + b) \times h$  where h is the height and a and b are the widths on the top and bottom
- Area of a parallelogram = base x vertical height
- Area of a circle =  $\pi r^2$
- The circumference of a circle is the perimeter distance around the edge
- Circumference of a circle =  $2\pi r$  or  $\pi d$
- Surface area is the area of all the faces of a 3D shape added together
- Perpendicular - at an angle of 90 degrees to a given line
- A compound shape is a shape made up of two or more 2D shapes put together to make a new shape

## HOW TO....

$$\begin{aligned} \text{Area} &= \frac{1}{2} \times \text{base} \times \text{perpendicular height} \\ &= \frac{1}{2} \times 11 \times 10 = 55 \text{ cm}^2 \end{aligned}$$



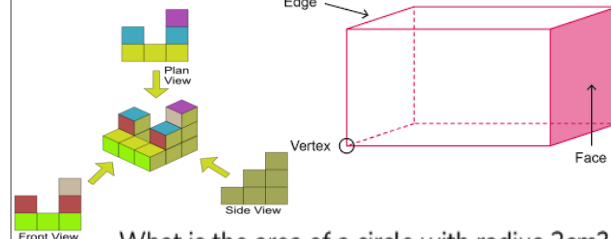
This cuboid is made from 24 unit cubes.

Its volume is

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

$$\text{Volume} = 2 \times 4 \times 3$$

$$\text{Volume} = 24 \text{ cm}^3$$



What is the area of a circle with radius 3cm?



$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= \pi \times 3^2 \\ &= 9\pi \text{ cm}^2 \\ &= 28.3 \text{ cm}^2 \text{ (1.d.p.)} \end{aligned}$$

## CORE

## GOOD TO KNOW...

## HOW TO....

- Recall times tables accurately e.g  $7 \times 8 = 56$  or  $9 \times 9 = 81$
  - For order of operations we use BIDMAS
    - Brackets
    - Indices
    - D/M divide and multiply
    - A/S add and subtract
  - A negative number is less than zero
    - Negative ( $\times/+$ ) negative gives a positive answer
    - Negative ( $\times/+$ ) positive gives a negative answer
    - ( $+ -$ ) means subtract - ( $- -$ ) means add
  - Understand place value - Place value is the value of each digit in a number. For example, the 5 in 350 represents 5 tens, or 50; however, the 5 in 5,006 represents 5 thousands, or 5,000
 

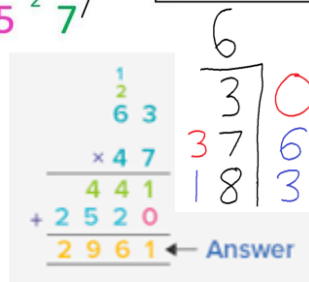
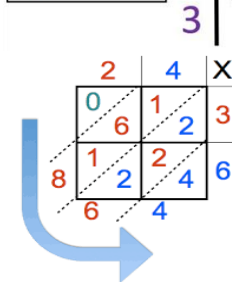
TH	H	T	U	.	$\frac{1}{10}$
3	7	8	9	.	6
  - Rounding - if 5 or more round up, if less than 5 stays the same
- Keywords:**
- Integer - A whole number ie. -1 or 4
  - Square number - The result of multiplying an integer by itself ie.  $3 \times 3 = 9$
  - Factor - a number that divides a number without a remainder ie. 5 is a factor of 10.
  - Multiple - The times tables of a number ie. 8 is a multiple of 2
  - Prime number - A number that has only two factors, 1 and itself ie. 11 is a prime number.
  - Decimal place - The amount of numbers after a decimal point. It is normally written as d.p.
  - Significant figure - The significant digits of a number are the digits that have meaning or contribute to the value. We start counting significant figures from the first non-zero figure. Ie. 0.086, the 8 is the first significant figure. It may be written as s.f.

- Cube numbers - A number multiplied by itself, then multiplied by itself again ie.  $2 \times 2 \times 2 = 8$
- Square root - Is the inverse of a square number ie. the square root of 16 is 4 as  $4 \times 4 = 16$
- Cube root - Is the inverse of a cube number ie. the cube root of 27 is 3 as  $3 \times 3 \times 3 = 27$
- Alternate meanings
  - Multiply is the same as times
  - Subtract is the same as take away
  - Product means multiply
  - Sum means add
- Inverse operations mean do the opposite
  - The inverse of add is subtract
  - The inverse of multiply is divide
- 1 is not a prime number as it only has one factor. 2 is the only even prime number.
- HCF - highest common factor
- To find the HCF of two or more numbers:
  - List all factors of both numbers
  - Find the highest factor that appears in both lists
- LCM - lowest common multiple
- To find the LCM of two or more numbers:
  - List multiples of both numbers
  - Repeat until you find the lowest common multiple
- Estimate - Rounding of a number to make a calculation easier ie. we can estimate  $99.6 \div 7.2$  to be  $100 \div 10 \approx 10$
- Round 4953 to 2 s.f.

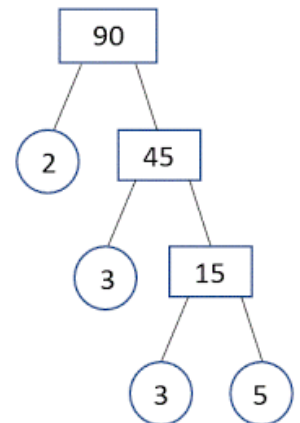
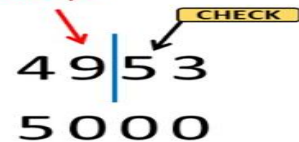
$$57 \div 3 = 19$$

How many times does 3 go into 5? It goes into 5 once and has a remainder of 2.

How many times does 3 go into 27? It goes into 27 nine times and has no remainder.



Stay or round up?



## CORE

## GOOD TO KNOW...

## HOW TO....

- A letter represents an unknown variable
- When multiplying powers add the powers  
e.g.  $6^4 \times 6^7 = 6^{11}$  OR  $a^3 \times a^5 = a^8$
- When dividing powers subtract the powers  
e.g.  $6^8 \div 6^5 = 6^3$  OR  $a^9 \div a^5 = a^4$
- When in brackets multiply the powers  
e.g.  $(8^4)^3 = 8^{12}$  OR  $(x^5)^2 = x^{10}$
- Inverse - opposite, e.g. inverse of add is subtract
- Term - a number of letter on its own e.g. 2 is a term, x is a term
- Expression - Numbers, symbols and operators grouped together e.g.  $2x + 3$  is an expression
- Indices - The power or exponent which is raised to a number or a variable. For example,  $2^4$ , 4 is the index of 2 and means  $2 \times 2 \times 2 \times 2$

- When simplifying indices, multiply both numbers together before multiplying both powers e.g.  
 $3x^3y \times 5x^2y^4 = 12x^5y^5$
- $3 \times 5 = 12$        $x^3 \times x^2 = x^5$        $y \times y^4 = y^5$
- When simplifying indices, divide both numbers before dividing both powers.  
 $\frac{12x^5y^3}{3x^2y} = 4x^3y^2$
- $12 \div 3 = 4$        $x^5 \div x^2 = x^3$        $y^3 \div y^1 = y^2$
- When simplifying indices, the number at the front of the term with a power needs to also be raised to that power  
 $(2a^2b)^3 = 8a^6b^3$
- $2 \times 2 \times 2 = 8$        $a^2 \times a^2 \times a^2 = a^6$        $b \times b \times b = b^3$

### Collecting Like Terms

**Ex1**  $x + 4y + 6x + 2y = 7x + 6y$

**Ex**  $3x + y - 2x + 4y = x + 5y$

### Laws of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

## CORE

## GOOD TO KNOW...

## HOW TO....

### Keywords

**Pie chart:** This is a type of graph in which a circle is divided into sectors that each represents a proportion of the whole.

**Mean:** Is the average of all the numbers. You add all the numbers up and divide them by the quantity of numbers.

**Median:** Is the middle number, when put in order of size. If there are two numbers in the middle, you add both together and divide by 2.

**Mode:** Is the most common number. This is the number that appears the most.

**Range:** Is the difference between the largest and smallest numbers in a set of data.

**Modal value:** The modal value of a set of data is the most frequently occurring value. It's a measure of central tendency that tells you the most popular/common choice of the sample.

**Estimated mean:** This is used to estimate the mean from a grouped data

**Two-way table:** It is a way of sorting data so that the frequency of each category can be seen quickly and easily.

### Averages from lists

#### The Mean

A measure of average to find the central tendency... a typical value that represents the data

24, 8, 4, 11, 8

Find the sum of the data (add the values)

55

Divide the overall total by how many pieces of data you have

$55 \div 5$

Mean = 11

#### The Mode (The modal value)

This is the number OR the item that occurs the most (it does not have to be numerical)

24, 8, 4, 11, 8

Mode = 8

This can still be easier if the data is ordered first

#### The Median

The value in the center (in the middle) of the data

Put the data in order 4, 8, 8, 11, 24

Find the value in the middle 4, 8, 8, 11, 24

Median = 8

NOTE: if there is no single middle value find the mean of the two numbers left

#### Range

Spread of the values

Difference between the biggest and smallest

3 9 8 12

Range: Biggest value - Smallest value

$12 - 3 = 9$

Range = 9

### Draw and interpret Pie Charts

Remember a circle has  $360^\circ$

Type of pet	Dog	Cat	Hamster
Frequency	32	25	3

$\frac{32}{60}$

"32 out of 60 people had a dog"

This fraction of the 360 degrees represents dogs

$\frac{32}{60} \times 360 = 192^\circ$



Use a protractor to draw This is  $192^\circ$

#### Multiple method

As 60 goes into 360 - 6 times Each frequency can be multiplied by 6 to find the degrees (proportion of 360)

Represents quantitative, discrete data

There were 60 people asked in this survey (Total frequency)

### Two way tables

60 people visited the zoo one Saturday morning 26 of them were adults 13 of the adult's favourite animal was an elephant 24 of the children's favourite animal was an elephant

Subgroups each have their own heading

	Adult	Child	Total
Elephant	13	24	37
Other	13	10	23
Total	26	34	60

Extract information to input to the two-way table

Needs subgroup totals

Overall total



## CORE

## GOOD TO KNOW...

## HOW TO....

### Keywords

**Whole numbers:** Are numbers that are not fractions or decimals and includes zero. Another name for whole number is an integer.

**Rounding:** To make a number simpler but keeping its value close to what it was.

**Decimal number:** Is a way of writing a number that is not whole. Decimal numbers fall between two whole numbers. For example, 12.5 is a decimal number between 12 and 13.

**Significant figure:** A digit that gives meaning to a number. The most significant digit (figure) in an integer is the number on the left. The most significant digit in a decimal fraction is the first non-zero number after the decimal point.

**Estimation:** Is a way of approximately calculating an answer to check its accuracy. A calculator is not needed when estimating, even with large numbers or decimals.

**Exchange rates:** This is the price of one currency expressed in terms of another currency. For example, £1 = £1.13 or £1 = \$1.20.

### Time

There are 60 seconds in a minute.

There are 60 minutes in an hour.

There are 24 hours in a day.

15 minutes can be expressed as 0.25 or  $\frac{1}{4}$  of an hour.

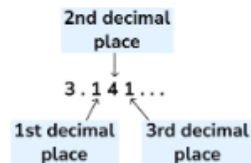
30 minutes can be expressed as 0.5 or  $\frac{1}{2}$  of an hour.

45 minutes can be expressed as 0.45 or  $\frac{3}{4}$  of an hour.

6 minutes can be expressed as 0.1 or  $\frac{1}{10}$  of an hour.

Decimal places are positions of the digits to the right of a decimal point.

E.g.



Estimation is when we use approximate values in a calculation to find an approximate answer.

When we estimate the numbers in a calculation, we usually round to 1 significant figure.

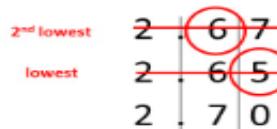
E.g.

Estimate  $5.7 \times 2.3 \rightarrow 5.7$  rounded to 1.s.f is 6  $\rightarrow$  So  $5.7 \times 2.3 \approx 6 \times 2 = 12$

2.3 rounded to 1.s.f is 2  $6 \times 2 = 12$

2.67    2.65    2.70

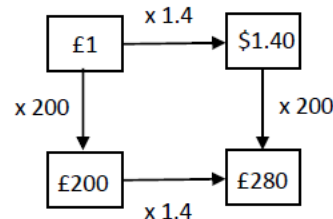
Starting with the most valuable column, compare digits until you find the lowest number.



2.65,    2.67,    2.70

Put these decimals in ascending order

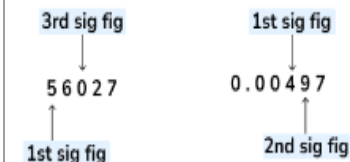
### Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

"Significant" means "important". The first significant figure (or significant digit) of a number is the most important digit which expresses the size of the number; it is the first non-zero digit.

E.g.





## CORE

## GOOD TO KNOW...

## HOW TO....

### Keywords

**Function:** A relationship that instructs how to get from an input to an output

**Input:** The number/symbol put into a function.

**Output:** The number/expression that comes out of a function.

**Inverse:** The operation that undoes what was done by the previous operation (the opposite operation).

**Substitute:** To replace one variable with a number or new variable.

**Expression:** A maths sentence with a minimum of two numbers and at least one maths operation without an equal sign.

**Equation:** This states that two things are equal. It will have an equals sign = to signify this.

**Inequality:** This compares two variables showing if one is greater than, less than or equal to another.

**Expand:** This means to multiply each term in the bracket by the expression outside the bracket.

**Factorise:** Is the reverse process of expanding brackets. To factorise an expression fully, means to put it in brackets by taking out the HCF of the terms in the expression.

**Variable:** A symbol for a number we don't know yet.

**Solution:** A value we can put in place of a variable that makes the equation true.

**Term:** A single number or variable.

**Formula:** A rule written with all mathematical symbols, eg area of a rectangle base x height.

### Solutions on a number line



$$x < 1$$

$$x \leq 1$$

$$x > 1$$

$$x \geq 1$$

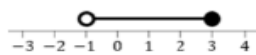
Both represent values less than 1

Both represent values more than 1

Includes the value 1

Includes the value 1

Values less than or equal to 3 but also more than -1



$$-1 < x \leq 3$$

This includes the integer values 0, 1, 2, 3

● Includes the value it sits above

○ Does NOT include the value it sits above

### Balancing method

$$8a - 5 = 11$$

$$+5 \quad +5$$

$$8a = 16$$

$$+8 \quad +8$$

$$a = 2$$

### Function machine method

$$8a - 5 = 11$$

$$a \rightarrow \times 8 \rightarrow -5 \rightarrow 11$$

$$2 \leftarrow +8 \leftarrow +5 \leftarrow 11$$

$$a = 2$$

### Expanding

$$2(g + 4)$$

$$= 2g + 8$$

### Multiply in

### Factorising

$$3x + 6 \equiv 3(x + 2)$$

Expanding brackets

## Form and solve inequalities



Two more than treble my number is greater than 11

Form

$$x \rightarrow \times 3 \rightarrow +2 \rightarrow 11$$

$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

$$x > 3$$

## CORE

Angles are measured in degrees  
 Angles on a straight line add up to 180  
 A right angle is 90 degrees  
 Angles around a point add up to 360  
 Angles in a triangle add up to 180  
 Angles in a quadrilateral add up to 360  
 Exterior angles add up to 360  
 Vertically opposite angles are equal  
 Co-interior angles add up to 180  
 Alternate angles are equal  
 Corresponding angles are equal

Parallel - Two straight lines equidistant apart which never meet

Polygon - a 2D shape made up of straight lines  
 Equilateral triangle - a triangle with 3 equal angles and sides  
 Isosceles triangle - a triangle with 2 equal angles and sides  
 Right angled triangle - a triangle with one right angle  
 Interior angle - an angle inside a polygon  
 Exterior - an angle outside a polygon

### Formula for sum of interior angles

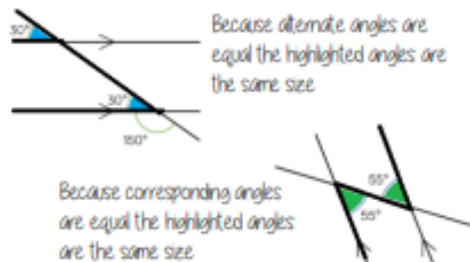
$$(n - 2) \times 180$$

### Exterior angles

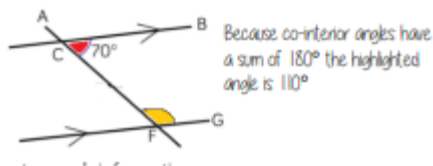
$$360 \div \text{number of sides of the shape}$$

## GOOD TO KNOW...

### Alternate/ Corresponding angles



### Co-interior angles



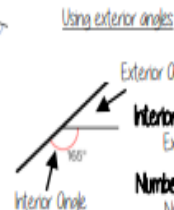
As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

- Names of 2D shapes
  - 5 sided - Pentagon
  - 6 sided - Hexagon
  - 7 sided - Heptagon
  - 8 sided - Octagon
  - 9 sided - Nonagon
  - 10 sided - Decagon

## HOW TO....

### Sum of exterior angles

Exterior angles all add up to 360°



Number of sides =  $360^\circ \div \text{exterior angle}$   
 Number of sides =  $360 \div 15 = 24$  sides

### Missing angles in regular polygons



$$\text{Exterior angle} = 360 \div 8 = 45^\circ$$

$$\text{Interior angle} = \frac{(8-2) \times 180}{8} = \frac{6 \times 180}{8} = 135^\circ$$

Exterior angles in regular polygons =  $360^\circ \div \text{number of sides}$

Interior angles in regular polygons =  $\frac{(\text{number of sides} - 2) \times 180}{\text{number of sides}}$

## CORE

**Percent:** parts per 100 – written using the % symbol.

**Decimal:** a number in our base 10 number system.

Numbers to the right of the decimal place are called decimals.

**Fraction:** a fraction represents how many parts of a whole value you have. Equivalent: of equal value

**Proportion:** numerical relationship that compares two things

**Numerator:** the number above the line on a fraction. The top number. Represents how many parts are taken

**Denominator:** the number below the line on a fraction.

The number represent the total number of parts.

**Whole:** a positive number including zero without any decimal or fractional parts

**Unit Fraction:** a fraction where the numerator is one and denominator a positive integer.

**Non-unit Fraction:** a fraction where the numerator is larger than one.

**Dividend:** the amount you want to divide up.

**Divisor:** the number that divides another number.

**Quotient:** the answer after we divide one number by another. e.g. dividend ÷ divisor = quotient

**Ratio:** a statement of how two numbers compare equal

**Parts:** all parts in the same proportion, or a whole shared equally

**Proportion:** a statement that links two ratios

**VAT:** Value added tax.

**Part:** a section of a whole

Equivalent: of equal value

## GOOD TO KNOW...

Convert FDP

$\frac{70}{100} \rightarrow$  This also means  $70 \div 100$   $\rightarrow$  70 out of 100 squares  $\rightarrow$  70 "hundredths" = 7 "tenths"  $\rightarrow$  0.7  $\rightarrow$  70 hundredths  $\rightarrow$  - 70%

Using a calculator  $\rightarrow$   $\frac{70}{100} = 0.7$

Convert to a decimal  $\rightarrow$   $\frac{70}{100} = 0.7$

Be careful of recurring decimals  
 eg  $\frac{1}{3} = 0.3333333$   
 $\frac{3}{10} = 0.3$   
 The dot above the 3

This will give you the answer in the simplest form  $\rightarrow$   $\times 100$  converts to a percentage

**Express as a % - Non-calculator** Percent – per hundred

$\frac{7}{10}$  } 7 per every 10 are orange } This means that 70 per every 100 are orange }  $\frac{70}{100}$  } 70%

$\frac{27}{50}$  } 27 per every 50 shaded }  $\frac{54}{100}$  } 54 per every 100 shaded } 54%

Denominator 100      Equivalent fractions

**Express as a % - Calculator**

Rose  $\frac{13}{30}$   $\rightarrow$   $\frac{13}{30} \times 100 = 43.3333...%$   $\rightarrow$  43%

This the same as  $13 \div 30$

Can't use equivalence easily to find 'per hundred'

Decimal percentages are still a percentage

## HOW TO....

Fraction/percentage of an amount

Find  $\frac{3}{5}$  of £60

$\frac{3}{5}$  of £60  $\rightarrow$  £36

Remember  $\frac{3}{5} = 60\%$

$10\%$  of £60 = £6  
 $50\%$  of £60 = £30  
 $60\%$  of £60 = £36

Remember  $\frac{3}{5} = 60\% = 0.6$   
 $0.6 \times 60 = 36$

**Percentage change**

I bought a phone for £200  
A year later sold it for £125

I bought a house for £180,000  
later sold it for £216,000

$\frac{125}{200} \times 100 = 62.5\%$   
 Percentage loss  $100 - 62.5 = 37.5\%$

$\frac{216000}{180000} \times 100 = 120\%$   
 Percentage profit  $120 - 100 = 20\%$

Difference in value  $\times 100$   
Original value

**Ratio In (or n:1)**

This is asking you to cancel down until the part indicated represents 1

Show the ratio 4:20 in the ratio of In

The question states that this part has to be 1 unit. Therefore Divide by 4

$4:20 \rightarrow 1:5$

This side has to be divided by 4 too – to keep in proportion

\*The n part does not have to be an integer for this type of question

## CORE

**Outcomes:** the result of an event that depends on probability

E.g. The outcome of rolling a dice would be landing on a two.

**Probability:** the chance that something will happen

**Set:** a collection of objects.

**Event:** the outcome of a probability – a set of possible outcomes

E.g. rolling a dice would be an event

**Biased:** a built in error that makes all values wrong by a certain amount.

**Union:** Notation 'U' meaning the set made by comparing the elements of two sets

Mutually exclusive events are events that can't both happen

**Probability:** concerning numerical descriptions of how likely an event is to occur

**Experimental probability:** Experimental probability is the ratio of the number of times an event occurs to the total number of trials or times the activity is performed.

$$P(A) + P(B) = 1$$

$$P(\text{not } A) = 1 - P(A)$$

**Theoretical probability:** the number of favorable outcomes divided by the total number of possible outcomes

**Independent event:** An event that is not affected by other events.

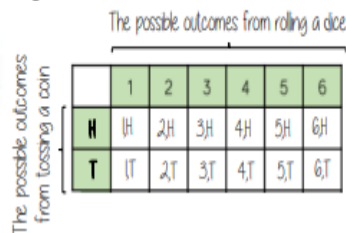
**Dependent event:** An event that is affected by previous events.

## GOOD TO KNOW...

### Construct sample space diagrams



Sample space diagrams provide a systematic way to display outcomes from events



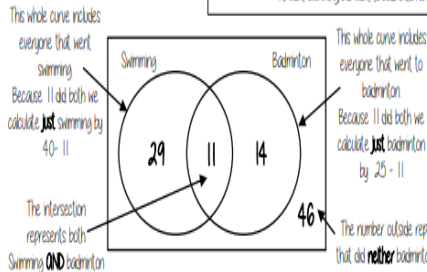
This is the set notation to list the outcomes S =

In between the { } are all the possible outcomes

$$S = \{ 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T \}$$

### Probability from Venn diagrams

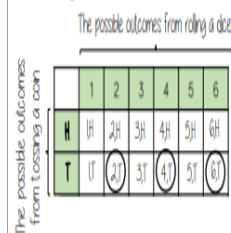
100 students were questioned if they played badminton or went to swimming club  
40 went swimming, 25 went to badminton and 11 went to both



$$P(\text{Just swimming}) = \frac{29}{100}$$

## HOW TO....

### Probability from sample space



This is the set notation that represents the question P

What is the probability that an outcome has an even number and a tails?

$$P(\text{Even number and Tails}) = \frac{3}{12}$$

In between the ( ) is the event asked for

There are three even numbers with tails

Numerator: the event

Denominator: the total number of outcomes

There are twelve possible outcomes

### Probability from two-way tables

	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

$$P(\text{Girl walk to school}) = \frac{21}{100}$$

The total number of items

The event

The total in the set

### The Probability of an Event NOT Occuring

$$P(\text{not } A) = 1 - P(A)$$

Ex: The probability of NOT tossing a 6 of a die.

$$\Omega = \{ 1, 2, 3, 4, 5, 6 \} \text{ (Sample space)}$$

$$A = \{ 6 \} \text{ (Event)}$$

$$P(A) = \frac{1}{6} \text{ (Probability of Event A)}$$

$$\text{therefore } P(\text{not } A) = 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$$

## CORE

## GOOD TO KNOW...

## HOW TO....

**Factor** - a number that divides a number without a remainder ie. 5 is a factor of 10.

**Multiple** - The times tables of a number ie. 8 is a multiple of 2

**Prime number** - A number that has only two factors, 1 and itself ie. 11 is a prime number.

- 1 is not a prime number as it only has one factor. 2 is the only even prime number.
- HCF - highest common factor
- To find the HCF of two or more numbers:
  - List all factors of both numbers
  - Find the highest factor that appears in both lists

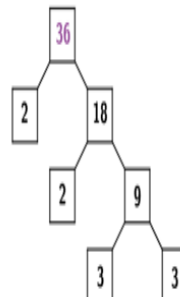
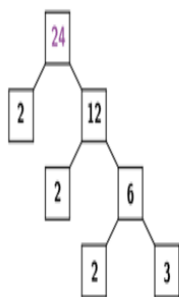
OR use prime factorisation and a Venn diagram

- LCM - lowest common multiple
- To find the LCM of two or more numbers:
  - List multiples of both numbers
  - Repeat until you find the lowest common multiple

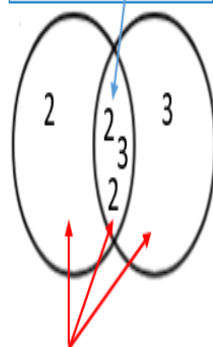
OR use prime factorisation and a Venn diagram

## HCF and LCM

Find the HCF and LCM of 24 and 36



$$\text{HCF: } 2 \times 2 \times 3 = 12$$



$$\text{LCM: } 2 \times 2 \times 2 \times 3 \times 3 = 72$$

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 20: 1, 2, 4, 5, 10, 20

HCF of 30 and 20: 10

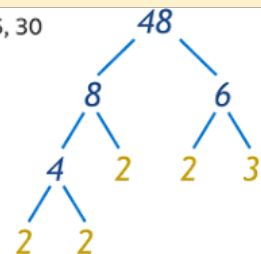
Find the Least Common Multiple

8, 4, 6

8 → 8, 16, 24, 32, 40, 48

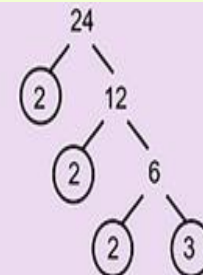
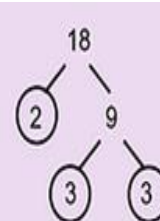
4 → 4, 8, 12, 16, 20, 24, 28, 32

6 → 6, 12, 18, 24, 30, 36



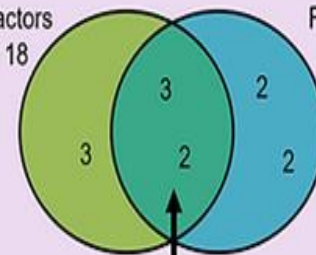
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Calculate the HCF and LCM of 18 and 24.



Factors of 18

Factors of 24



Factors of both

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 3 \times 2 \times 3 \times 2 \times 2 = 72$$



## CORE

## GOOD TO KNOW...

## HOW TO....

**Area:** Space inside a 2D object

**Perimeter:** Length around the outside of a 2D object

**Pi ( $\pi$ ):** The ratio of a circle's circumference to its diameter

**Formula:** A mathematical relationship/ rule given in symbols.

E.g.  $b \times h = \text{area of rectangle/ square}$

**Infinity ( $\infty$ ):** A number without a given ending (too great to count to the end of the number) – never ends

**Sector:** A part of the circle enclosed by two radii and an arc.

**Circumference** The distance around the outside of a circle

**Diameter:** a straight line passing from side to side through the centre of a circle

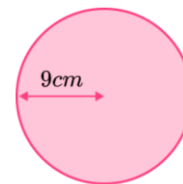
**Radius:** a straight line from the centre to the circumference of a circle

### Area of a circle (Calculator)



How to get  $\pi$  symbol on the calculator

Find the area of this circle. Give your answer to 1 decimal place.



**1** Find the radius or diameter of the circle.

To find the area, we need to know the radius. The radius of this circle is 9 cm.

**2** Use the relevant formula.

The circle formula for area is  $A = \pi r^2$ .

$$A = \pi r^2$$

$$A = \pi \times 9^2$$

$$= 254.4690049$$

**3** Give your answer clearly with the correct units.

We need to give our answer to 1 decimal place. Since the radius is measured in cm, the area will be measured in  $\text{cm}^2$ .

$$\text{Area} = 254.5\text{cm}^2 \text{ (1dp)}$$

### Common misconceptions

- Incorrectly using the radius or diameter of a circle

It is important to know the difference between the diameter and the radius of a circle and to use the correct one in the calculation.

- Using an incorrect formula

It can be easy to get confused between the different formulae – make sure you know which formula is for which calculation.

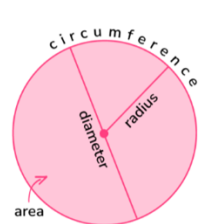
- Incorrect use of BIDMAS

When calculating the area of a circle, we need to calculate the square of the radius first as the radius is raised to the power of 2, and then multiply this value by  $\pi$ . This is because indices come before multiplication in BIDMAS.

## Area and Circumference of a Circle

The **area** of a circle is the amount of space within a circle.

The **circumference** of a circle is the distance around the edge of the circle.

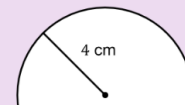
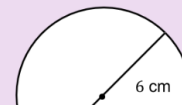


$$\begin{aligned} \text{Circumference} &= \pi d = 2\pi r \\ \text{Area} &= \pi r^2 \end{aligned}$$

The radius is half of the diameter of a circle.

When you divide the circumference by the diameter for **any circle**, you **always** get  $\pi$  (approximately 3.142 to 3dp)

Calculate the area and circumference of these circles:



## CORE

## GOOD TO KNOW...

## HOW TO....

**Numerator** : the number above the line on a fraction. The top number. Represents how many parts are taken

**Denominator**: the number below the line on a fraction. The number represent the total number of parts

**Equivalent**: of equal value

**Mixed numbers**: a number with an integer and a proper fraction

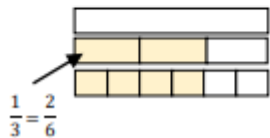
**Improper fractions**: a fraction with a bigger numerator than denominator

**Lowest Common Denominator**: The Lowest Common Multiple of both the denominators of fractions being added or subtracted

### Equivalent fractions

Numerator and denominator have the same multiplier

$$\frac{2}{3} = \frac{4}{6}$$



To simplify or cancel down fractions, we need to find an equivalent fraction with the smallest

### How to convert a mixed number to improper fraction

In order to convert a mixed number to an improper fraction:

- 1 Multiply the whole number by the denominator.
- 2 Add on the numerator.

### How to convert improper fractions to mixed numbers

In order to change an improper fraction to a mixed number:

- 1 Work out how many times the denominator divides into the numerator.
- 2 Work out the remainder.
- 3 Write the mixed number with the whole number at the front and the remainder as the new numerator over the original denominator.

$$\frac{3}{4} \text{ of } 36$$

As we are asked to work out three quarters of 36, let's start by working out one quarter:

$$\frac{1}{4} \text{ of } 36 = 9$$

So to work out three quarters we multiply this by 3:

$$\frac{3}{4} \text{ of } 36 = 27$$

### Add/Subtraction any fractions



Use equivalent fractions to find a common multiple for both denominators

$$\frac{1}{2} \times \frac{1}{3}$$

- 1 Multiply the numerators together:  $1 \times 1 = 1$
- 2 Multiply the denominators together:  $2 \times 3 = 6$
- 3 Simplify if possible:  $\frac{1}{6}$ .

### Dividing any fractions *Remember to use reciprocals*

$$\frac{2}{5} \div \frac{3}{4}$$

Multiplying by a reciprocal gives the same

Represented

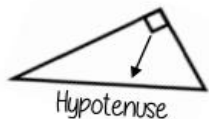


$$= \frac{8}{15}$$

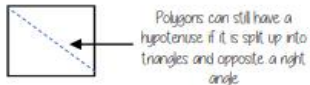
## CORE

**Square number:** the output of a number multiplied by itself  
**Square root:** a value that can be multiplied by itself to give a square number  
**Hypotenuse:** the largest side on a right angled triangle.  
 Always opposite the right angle.  
**Opposite:** the side opposite the angle of interest  
**Adjacent:** the side next to the angle of interest  
**Quadrant:** four quarters of the coordinate plane.  
**Coordinate:** a set of values that show an exact position.  
**Horizontal:** a straight line from left to right (parallel to the x axis)  
**Vertical:** a straight line from top to bottom (parallel to the y axis)  
**Origin:** (0,0) on a graph. The point the two axes cross

### Identify the hypotenuse



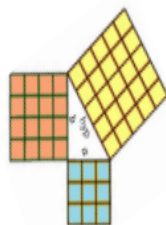
The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle.

## GOOD TO KNOW...

### Determine if a triangle is right-angled



$$a = 3 \quad b = 4 \quad c = 5$$

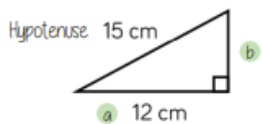
If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$\begin{aligned} \text{eg } a^2 + b^2 &= \text{hypotenuse}^2 \\ 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \end{aligned}$$

Substituting the numbers into the theorem shows that this is a right-angled triangle

### Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$

1 Substitute in the values you are given

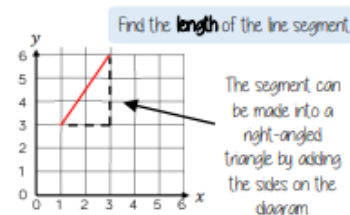
$$\begin{aligned} 144 + b^2 &= 225 \\ -144 & \quad -144 \end{aligned}$$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

$$\begin{aligned} \text{Square root to find the length of the side} \quad & \left\{ \begin{aligned} b^2 &= 111 \\ b &= \sqrt{111} = 10.54 \text{ cm} \end{aligned} \right. \end{aligned}$$

## HOW TO....

### Pythagoras' theorem on a coordinate axis



The segment can be made into a right-angled triangle by adding the sides on the diagram

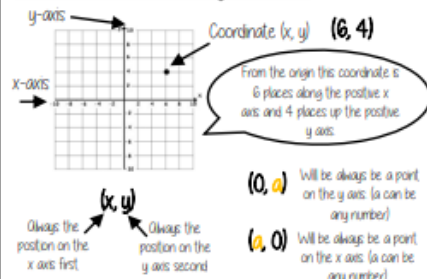
The line segment is the hypotenuse

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle

Be careful to check the scale on the axes

### Coordinates in four quadrants



**(0, a)** Will always be a point on the y axis. (a can be any number)  
**(a, 0)** Will always be a point on the x axis. (a can be any number)

Always the position on the x axis first  
 Always the position on the y axis second



## CORE

**Variable:** a quantity that may change within the context of the problem.

**Relationship:** the link between two variables (items). E.g. Between sunny days and ice cream sales

**Correlation:** the mathematical definition for the type of relationship..

**Origin:** where two axes meet on a graph.

**Line of best fit:** a straight line on a graph that represents the data on a scatter graph.

**Outlier:** a point that lies outside the trend of graph.

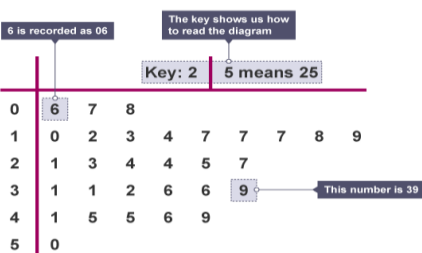
**Stem and Leaf Diagram** - Shows numerical data split into "leaves" (usually the last digit) and a "stem" (the other digits).

**Mean:** the average of the given numbers and is calculated by dividing the sum of given numbers by the total number of numbers.

**Mode:** the value that appears most frequently in a data set.

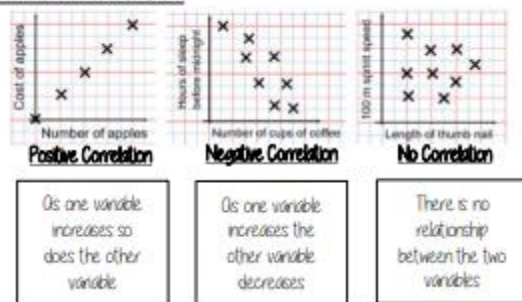
**Median:** the middle number in a sorted, ascending or descending list of numbers

**Range:** The difference between the lowest and



## GOOD TO KNOW...

### Linear Correlation

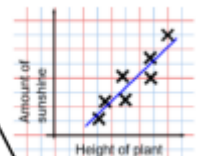


### The line of best fit

The Line of best fit is used to make estimates about the information in your scatter graph

#### Things to know

- The line of best fit **DOES NOT** need to go through the origin (The point the axes cross)
- There should be approximately the same number of points above and below the line. It may not go through any points)
- The line extends across the whole graph



It is only an estimate because the line is designed to be an average representation of the data

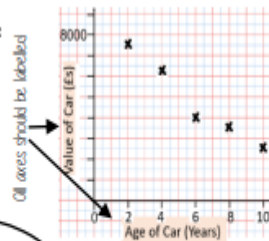
It is always a **straight line**.

## HOW TO....

### Draw and interpret a scatter graph

Age of Car (Years)	2	4	6	8	10
Value of Car (£)	7500	6250	4000	3500	2500

- This data may not be given in size order
- The data forms information pairs for the scatter graph
- Not all data has a relationship



The link between the data can be explained verbally

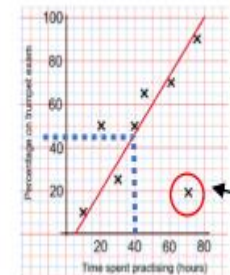
"This scatter graph shows as the age of a car increases the value decreases"

The axes should fit all the values on and be equally spread out

### Using a line of best fit

**Interpolation** is using the line of best fit to estimate values inside our data point

e.g. 40 hours revising predicts a percentage of 45



**Extrapolation** is where we use our line of best fit to predict information outside of our data

\*\*This is not always useful - in this example you cannot score more than 100% So revising for longer can not be estimated\*\*

This point is an **'outlier'** it is an outlier because it doesn't fit this model and stands apart from the data

## CORE

## GOOD TO KNOW...

## HOW TO....

Mirror line: a line that passes through the center of a shape with a mirror image on either side of the line  
 Line of symmetry: same definition as the mirror line

Horizontal: a straight line from left to right (parallel to the x axis)

Vertical: a straight line from top to bottom (parallel to the y axis)

Symmetry: when two or more parts are identical after a transformation.

Vertex: a point two edges meet.

- Transformations - Transformations change the size or position of shapes. There are four types of transformations: reflections, enlargements, rotations, translations.
- Reflection - A shape can be reflected across a line of reflection to create an image, like looking in a mirror. The line of reflection is also called the mirror line. Every point in the image is the same distance from the mirror line as the original shape.
- Rotation - Rotation turns a shape around a fixed point called the centre of rotation. There are three things needed to rotate a shape: the centre of rotation (a coordinate), the angle of rotation ( $90^\circ$ ,  $180^\circ$  etc.) and the direction of rotation (clockwise or anti-clockwise)
- Translation - A translation moves a shape up, down or from side to side but it does not change its appearance in any other way.
- Enlargement - Enlarging a shape changes its size. The shape can get either bigger or smaller.

### TRANSFORMATIONS

A CHANGE IN THE POSITION OR SIZE OF AN OBJECT

**TRANSLATION**  
 Described by a vector

VECTOR  $\begin{pmatrix} 7 \\ 1 \end{pmatrix}$

**REFLECTION**  
 Described by a mirror line

MIRROR LINE  $x = 6$

**ROTATION**  
 Described by an angle and a centre

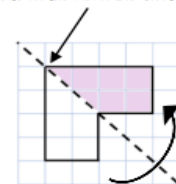
$90^\circ$  CLOCKWISE, CENTRE (6, 1)

**ENLARGEMENT**  
 Described by a scale factor and a centre

SCALE FACTOR = 2, CENTRE (1, 2)

### Reflect Diagonally (1)

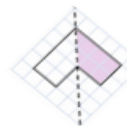
Points on the mirror line don't change position



Fold along the line of symmetry to check the direction of the reflection

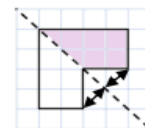
### Turn your image

If you turn your image, it becomes a vertical/ horizontal reflection (also good to check your answer this way)



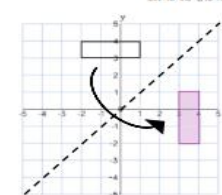
### Drawing perpendicular lines

Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

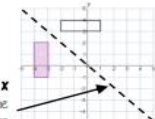


### Reflect Diagonally (2)

This is the line  $y = x$  (every y coordinate is the same as the x coordinate along this line)



This is the line  $y = -x$  The x and y coordinate have the same value but opposite sign



### Turn your image

If you turn your image, it becomes a vertical/ horizontal reflection (also good to check your answer this way)



### Rotate from a point (in a shape)

Original shape

Point of rotation

Image  $90^\circ$  clockwise

- 1 Trace the original shape (mark the point of rotation)
  - 2 Keep the point in the same place and turn the tracing paper
  - 3 Draw the new shape
- 

### Rotate from a point (outside a shape)

Original shape

Point of rotation

Image  $90^\circ$  anti-clockwise

- 1 Trace the original shape (mark the point of rotation)
- 2 Keep the point in the same place and turn the tracing paper
- 3 Draw the new shape



# Knowledge Organisers

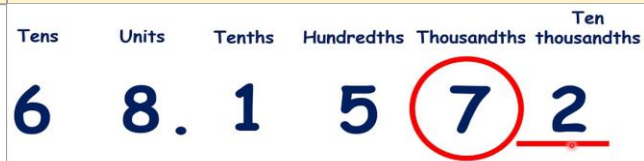
## Year 9

## CORE

## GOOD TO KNOW...

## HOW TO....

- The priority of operations is: brackets, indices, division, multiplication, addition and subtraction. This is called BIDMAS
  - When there is only addition and subtraction or multiplication and division on the same line, we work from left to right.
- Finding the square root is the inverse of finding the square
- Finding the cube root is the inverse of finding the cube
- To round a number to 1 decimal place (1 d.p.), look at the digit in the second decimal place. If it is 5 or more, round up.
- To multiply decimals, ignore the decimal and work out the normal calculation, then put the number of total digits after the decimal place in the question back into the answer. i.e.  $1.1 \times 1.2 = 1.32$
- To divide by a decimal, multiply both numbers by a power of ten (10, 100, 1000 etc.) until you have a whole number to divide by. Then work out the division using the bus stop method.
- A factor is a number that goes into another number without leaving a remainder i.e. 5 is a factor of 20 as 5 goes into 20 four times.
- A multiple is the times tables of a number i.e. the first three multiples of 6 are 6, 12 and 18.



**Round 68.1572 to the nearest:**

Whole number: **68**      2 decimal places: **68.16**

1 decimal place: **68.2**      3 decimal places:

Round <b>7.82438</b> to 3 significant figures	→	<b>7.82</b>
Round <b>4537</b> to 1 significant figure	→	<b>5 000</b>
Round <b>37.85672</b> to 3 significant figures	→	<b>37.9</b>
Round <b>6973</b> to 2 significant figures	→	<b>7000</b>

### Laws of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

Factors of 30: 1, 2, 3, 5, 6, **10**, 15, 30

Factors of 20: 1, 2, 4, 5, **10**, 20

HCF of 30 and 20: **10**

Find the Least Common Multiple

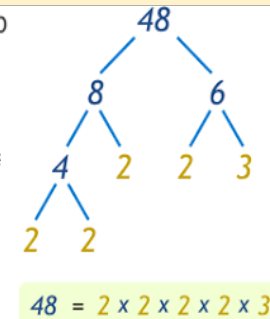
8, 4, 6

8 → 8, 16, **24**, 32, 40, 48

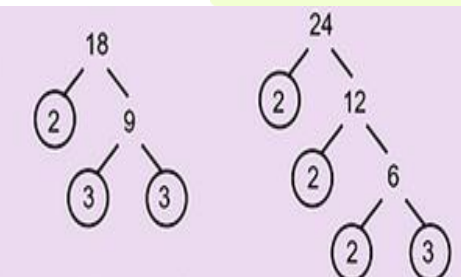
4 → 4, 8, 12, 16, 20, **24**, 28, 32

6 → 6, 12, 18, **24**, 30, 36

**48 = 2 x 2 x 2 x 2 x 3**



Calculate the HCF and LCM of 18 and 24.



Factors of 18: 2, 3, 3, 9

Factors of 24: 2, 2, 2, 3, 6, 12

Factors of both: 2, 3, 6

HCF =  $2 \times 3 = 6$

LCM =  $3 \times 2 \times 3 \times 2 \times 2 = 72$

## CORE

## GOOD TO KNOW...

## HOW TO....

- The priority of operations is: brackets, indices, division, multiplication, addition and subtraction. This is called BIDMAS
  - When there is only addition and subtraction or multiplication and division on the same line, we work from left to right.
- Finding the square root is the inverse of finding the square
- Finding the cube root is the inverse of finding the cube
- To round a number to 1 decimal place (1 d.p.), look at the digit in the second decimal place. If it is 5 or more, round up.
- A factor is a number that goes into another number without leaving a remainder i.e. 5 is a factor of 20 as 5 goes into 20 four times.
- A multiple is the times tables of a number i.e. the first three multiples of 6 are 6, 12 and 18.

### Indices

-  $2 \times 2 \times 2 \times 2$  can be written  $2^4$

### Standard Form

$$a \times 10^n$$

Where  $1 \leq a < 10$  and  $n$  is an integer.

If  $n$  is positive, multiply 'a' by  $10$  'n' times.

If  $n$  is negative, divide 'a' by  $10$  'n' times (this will decrease the value and be a decimal).

- When there are  $m$  ways of doing one task and  $n$  ways of doing another, the total number of ways of doing the first task and then the second task is  $m \times n$  ways.
- A factorial is the result of multiplying a sequence of descending integers. i.e.  $4! = 4 \times 3 \times 2 \times 1 = 24$

## Negative Exponents

$$a^{-n} = \frac{1}{a^n} \quad \text{For } a \neq 0$$

$a^{-n}$  is a reciprocal of  $a^n$

Example:

$$3^{-2} = \frac{1}{3^2}$$

$$\left(\frac{2}{5}\right)^{-6} = \left(\frac{5}{2}\right)^6$$

## Fractional Indices

Numerator - Power

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

Denominator - Root

Examples:

$$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

$$25^{\frac{3}{2}} = \left(\sqrt[2]{25}\right)^3 = 5^3 = 125$$

## Laws of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 20: 1, 2, 4, 5, 10, 20

HCF of 30 and 20: 10

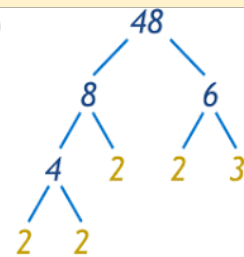
Find the Least Common Multiple

8, 4, 6

8 → 8, 16, 24, 32, 40, 48

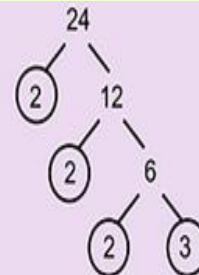
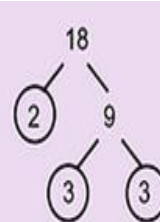
4 → 4, 8, 12, 16, 20, 24, 28, 32

6 → 6, 12, 18, 24, 30, 36



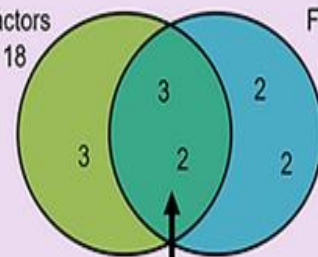
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Calculate the HCF and LCM of 18 and 24.



Factors of 18

Factors of 24



Factors of both

$$\text{HCF} = 2 \times 3 = 6$$

$$\text{LCM} = 3 \times 2 \times 3 \times 2 \times 2 = 72$$



## CORE

## GOOD TO KNOW...

## HOW TO....

- A term is a number, letter, or a number and a letter multiplied together i.e.  $x$ ,  $3a$ ,  $7y^2$  are all terms
- 'Like terms' contain the same letter to the same power (or contain no letters at all). You can simplify expressions by collecting like terms. i.e.  $2x + 3x = 5x$
- Terms can be simplified when multiplying or dividing even when they are not like terms. i.e.  $a \times b = ab$ 
  - When multiplying, write the letters in alphabetical order
  - Write the number before the letter(s)
- Substitution means putting numbers in place of letters.
- The factors of a term are all of the numbers and letters that divide exactly into it.
- A common factor is a factor of two or more terms.
- Expand - multiply term outside the bracket by all terms inside the brackets to eliminate brackets

### Laws of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

Terms can be simplified when multiplying or dividing, even when they are not like terms.

$$a \times b = ab \quad x \div y = \frac{x}{y}$$

When multiplying:

- write letters in alphabetical order
- write numbers before letters

Evaluate  $3a - 2b$ , for  $a = 10$  and  $b = 4$

$$\begin{aligned} 3a - 2b & \quad (a = 10 \quad b = 4) \\ &= 3(10) - 2(4) \\ &= 30 - 8 \\ &= 22 \quad \checkmark \end{aligned}$$

Like Terms	Unlike Terms	Factorise	Answer
$2x + 19x$	$2x + 19a$	$7x + 14$	$7(x + 2)$
$4w - 10w$	$4w - 10w^2$	$45 - 27k$	$9(5 - 3k)$
$14.2r - 12r$	$12r - 12s$	$12ab + 7b$	$b(12a + 7)$
$32a^2 + 9a^2$	$32a^2 + 9a^3$	$y^2 - 9y$	$y(y - 9)$
$8y + 5y$	$8y + 5$	$8t - 32t^2$	$8t(1 - 4t)$

## Collecting Like Terms

Ex1  $x + 4y + 6x + 2y = 7x + 6y$

Ex  $3x + y - 2x + 4y = x + 5y$

$3(a+4) = 3a + 12$

$4(a-5) = 4a - 20$

Factorising

$3x + 6 = 3(x + 2)$

Expanding brackets

Expand & Simplify...

$5(x+3) + 6(x-4)$

$5x + 15 + 6x - 24$   
 $11x - 9$

1)  $3a + 6y = 3(a + 2y)$

$4x + 32 = 4(x + 8)$

## CORE

## GOOD TO KNOW...

## HOW TO....

### Indices

-  $2 \times 2 \times 2 \times 2$  can be written  $2^4$

- When multiplying powers add the powers  
e.g.  $6^4 \times 6^7 = 6^{11}$  OR  $a^3 \times a^5 = a^8$
- When dividing powers subtract the powers  
e.g.  $6^8 \div 6^5 = 6^3$  OR  $a^9 \div a^5 = a^4$
- When in brackets multiply the powers  
e.g.  $(8^4)^3 = 8^{12}$  OR  $(x^5)^2 = x^{10}$
- Any number to the power of zero is 1

- Expand - multiply term outside the bracket by all terms inside the brackets to eliminate brackets
- The factors of a term are all of the numbers and letters that divide exactly into it.
- A common factor is a factor of two or more terms.
- The subject of a formula is the letter on its own, on one side of the equals sign.

- A term is a number, letter, or a number and a letter multiplied together i.e.  $x$ ,  $3a$ ,  $7y^2$  are all terms
- An expression contains letter and/ or number terms but no equal sign.
- An equation has an equals sign, letter terms and numbers. You can solve it to find the value of the letter.
- An identity is true for all values of letters
- A formula has an equals sign and letters to represent different quantities. The letters are variables as their values can vary.

Q1) Expand:  $(x + 3)(x - 2)$

	$x$	$-2$
$x$	$x^2$	$-2x$
$+3$	$+3x$	$-6$

$$(x + 3)(x - 2) = x^2 + x - 6$$

Factorise	Answer
$7x + 14$	$7(x + 2)$
$45 - 27k$	$9(5 - 3k)$
$12ab + 7b$	$b(12a + 7)$
$y^2 - 9y$	$y(y - 9)$
$8t - 32t^2$	$8t(1 - 4t)$
$16gh + 28gf$	$4g(4h + 7f)$
$21w^2z - 77wx$	$7w(3wz - 11x)$

### Expand & Simplify...

$$5(x + 3) + 6(x - 4)$$

$$5x + 15 + 6x - 24$$

$$11x - 9$$

### Finding nth term of linear sequence

1) 6, 10, 14, 18, 22

The sequence increases by 4, so the nth term starts with  $4n$

Now compare the sequence to the 4 times table

6, 10, 14, 18, 22

Each term is 2 bigger than the 4 times table

So the nth term is  $4n + 2$

a Make  $a$  the subject of the formula  $v^2 = u^2 + 2as$

b Make  $x$  the subject of the formula  $y = \frac{ax + b}{c}$

1a  $v^2 = u^2 + 2as$

b  $y = \frac{ax + b}{c}$

$v^2 - u^2 = 2as$  - Subtract  $u^2$  from both sides.

$cy = ax + b$  - Multiply both sides by  $c$ .

$\frac{v^2 - u^2}{2s} = a$  - Divide both sides by  $2s$ .

$cy - b = ax$  - Subtract  $b$  from both sides.

$a = \frac{v^2 - u^2}{2s}$  - Re-write in the form  $a = \dots$

$\frac{cy - b}{a} = x$  - Divide both sides by  $a$ .

$x = \frac{cy - b}{a}$  - Re-write in the form  $x = \dots$

## CORE

**Graphs, tables and charts** are used to display, interpret and compare data.

**Discrete Data** - Can only have particular values, e.g shoe size.

**Continuous Data** - Measured and can have any values e.g length and time.

**Grouped Frequency Table** - Contains sorted data in groups called classes.

**Two-way Table** - Divides data into groups in rows across and in columns down the table. You can calculate totals across and down.

**Stem and Leaf Diagram** - Shows numerical data split into "leaves" (usually the last digit) and a "stem" (the other digits).

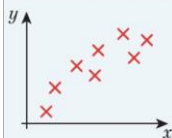
**Pie Chart** - A circle divided into sectors, each sector represents a set of data.

**Scatter Graphs** - Shows the relationship between two sets of data. Plot the points with crosses. Do not join them up.

- **Correlation:** Relationship between the sets of data.
- **Outlier:** A value that does not fit the pattern.
- **Line of best fit:** A straight line drawn through the middle of the points representing the trend.

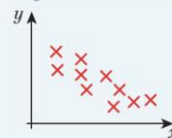
## GOOD TO KNOW...

**Positive correlation**



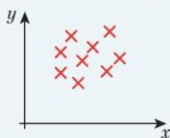
As  $x$  increases  
 $y$  increases

**Negative correlation**



As  $x$  increases  
 $y$  decreases

**No correlation**



No relationship  
between  $x$  and  $y$

	Baseball	Basketball	Football	Total
Male	13	15	20	48
Female	23	16	13	52
Total	36	31	33	100

11, 4, 27, 18, 18, 3, 24, 22, 11, 22, 18, 11, 18, 7, 29, 18, 11, 6, 29, 11

Intervals	Tally Marks	Frequency
0 - 5		2
5 - 10		2
10 - 15		5
15 - 20		5
20 - 25		3
25 - 30		3

Here,

0-5, 5-10, 10-15, ..... are class Intervals

## HOW TO....

Only place the last digit of each number in the 'leaf'

Arrange the numbers from smallest to largest

101, 131, 114, 102, 125, 101, 115, 103, 120, 122

stem | leaf

10 | 1 1 2 3  
11 | 4 5  
12 | 0 2 5  
13 | 1

103 is represented with a '3' in the '10' stem

Key:

10 | 3 = 103

Place all other digits of the number in the 'stem'

The table shows the match results of a football team.  
Draw a pie chart to represent the data.

Result	Won	Drawn	Lost
Frequency	28	12	20

Total number of games =  $28 + 12 + 20 = 60$

$$\begin{aligned} &\div 60 \quad 60 \text{ games} : 360^\circ \quad \div 60 \\ &\quad \quad \quad 1 \text{ game} : 6^\circ \end{aligned}$$

The total number of games is the total frequency.

$$1 \text{ game} = 360 \div 60 = 6^\circ$$

Work out the angle for one game.

$$\text{Won: } 28 \times 6^\circ = 168^\circ$$

Work out the angle for each result.

$$\text{Drawn: } 12 \times 6^\circ = 72^\circ$$

$$\text{Lost: } 20 \times 6^\circ = 120^\circ$$

Check that your angles total  $360^\circ$ .

$$\text{Check: } 168 + 72 + 120 = 360$$

Team results



Draw the pie chart. Give it a title and label each section, or make a key.



# Year 9 Term 3 Maths Knowledge Organiser [H unit 3-Interpreting&representing data] TKAT

## CORE

**Stem and Leaf Diagram** - Shows numerical data split into "leaves" (usually the last digit) and a "stem" (the other digits).

**Frequency Polygon** - To draw a frequency polygon, plot the frequency against the midpoints for each group.

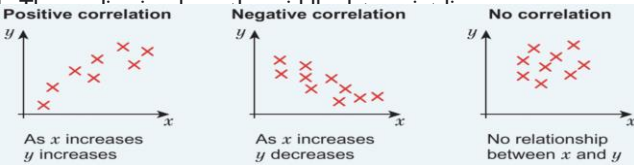
**Time-series Graphs** - A time-series graph is a line graph with time plotted on the horizontal axis.

**Scatter Graphs** - Shows the relationship between two sets of data. Plot the points with crosses. Do not join them up.

- **Correlation:** Relationship between the sets of data.
- **Outlier:** A value that does not fit the pattern.
- **Line of best fit:** A straight line drawn through the middle of the points representing the trend.

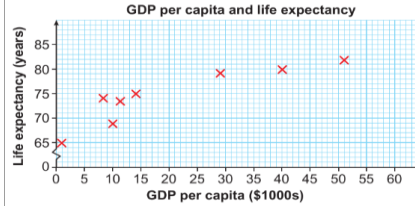
## Averages and Range

The modal class has the highest frequency. Make sure to write down the class and not the frequency.

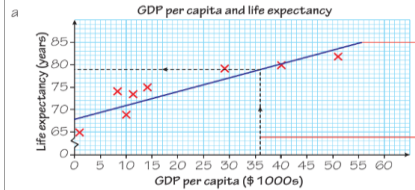


## GOOD TO KNOW...

The scatter graph shows the GDP per capita (in \$1000s) and life expectancy (in years) for eight countries.



- a Draw a line of best fit.  
b The GDP per capita in the UK is \$36,000. Estimate the life expectancy of a baby born in the UK.



Position a transparent ruler over your scatter graph so it follows the overall trend. Move it slightly so you have roughly the same number of points above and below the line.

To estimate life expectancy, start at \$36,000 on the horizontal axis, go up to the line of best fit and read off the answer on the vertical axis.

- b Estimated life expectancy in the UK is 79 years.

The annual salaries of employees working in an ICT company are displayed in the back-to-back stem and leaf diagram.

Key	Male	Female							
8   1	represents a salary of £18,000	1   9	represents a salary of £19,000						
	Male	Female							
	8	1	9						
	9	5	2	0	2	1	2	6	7
	8	7	3	0	3	0	4	4	
				4	5	6			
				5	4	8			

Compare the distribution of salaries of the male and female employees.

Male range:  $38,000 - 18,000 = £20,000$

Female range:  $58,000 - 19,000 = £39,000$

There are 9 males, so median male salary is:  $\frac{9+1}{2} = 5$ th value = £29,000

There are 13 females so median female salary is:  $\frac{13+1}{2} = 7$ th value = £30,000

Female employees' salaries have a larger range but the median salaries of men and women are similar.

Write a sentence comparing ranges and medians.

## HOW TO....

The table shows the times,  $T$ , taken for 100 people to queue for a rollercoaster at a theme park

- a Estimate the mean waiting time.  
b Explain why the mean is only an estimate.

The third column gives an estimate of the waiting time in each class.

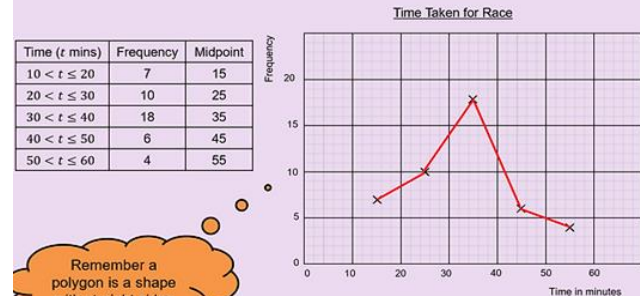
Time, $T$ (mins)	Frequency, $f$	Class midpoint, $x$	$xf$
$0 \leq T < 20$	14	10	$10 \times 14 = 140$
$20 \leq T < 40$	55	30	$30 \times 55 = 1650$
$40 \leq T < 60$	31	50	$50 \times 31 = 1550$
<b>Total</b>	100		3340

The fourth column gives an estimate of the total waiting time in each class.

$$\text{Mean} = \frac{\text{sum of waiting times}}{\text{total number of people}} = \frac{3340}{100} = 33.4 \text{ minutes}$$

- b The mean is an estimate because we don't know the exact times taken.


A frequency polygon can be drawn directly from the frequency table by using by finding the midpoint of each class interval.



Remember a polygon is a shape with straight sides

CORE	GOOD TO KNOW...	HOW TO....
<p><b>Operations with Fractions</b></p> <ul style="list-style-type: none"> <li>- <b>Add/ subtract</b> fractions by finding equivalent fractions with the same denominator</li> <li>- <b>Multiply</b> fractions by multiplying the numerators together and the denominators together</li> <li>- <b>Divide</b> fractions by following the KFC rule: keep the first fraction as it is, flip the second fraction around so the numerator becomes the denominator and change the sign from a divide to times.</li> </ul> <p><b>Finding Percentages</b></p> <p>50% - Divide amount by 2                      10% - Divide amount by 10                      1% - Divide amount by 100</p> <p><b>Keywords</b></p> <p><b>Fraction</b> - A fraction represents a part of a whole.  <b>Decimal</b> - A number with a decimal point in it.  <b>Percentage</b> - A part of a whole expressed in hundredths. e.g 1% of £100 = £1  <b>Numerator</b> - The part of a fraction that is above the line and signifies the number to be divided by the denominator.  <b>Denominator</b> - The part of a fraction that is below the line and that functions as the divisor of the numerator.  <b>Simple interest</b> - is the interest calculated only on the original amount invested. It is the same each year.</p>	<p><b>Mixed Number</b> - A number consisting of a whole number and a proper fraction.</p> <p><b>Improper Fraction</b> - A fraction whose numerator is larger than the denominator.</p> <p>Find the simple interest when £5000 is invested at 2.75% per annum over 2 years.</p> <p>2.75% = 0.0275 ——— Convert the percentage to a decimal multiplier.</p> <p>5000 × 0.0275 = £137.50 ——— This is the interest earned over 1 year.</p> <p>£137.50 × 2 = £275 ——— Multiply your answer by 2.</p> <p><b>Communication hint</b>                      Per annum or p.a. means 'each year'.</p> <p>There are 20 students in a class. 6 are male. What percentage of the class is male?</p> <p>Method A: <math>\frac{6}{20} \times 100\% = 6 \times \frac{100}{20}\% = 30\%</math></p> <p>Method B: <math>\frac{6}{20} \times \frac{5}{5} = \frac{30}{100} = 30\%</math> ——— Convert to a fraction with denominator 100.</p> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Work out</p> <math display="block">\frac{3}{4} \times \frac{2}{7} = \frac{3 \times 2}{4 \times 7} = \frac{6}{28} = \frac{3}{14}</math> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Work out</p> <math display="block">\frac{3}{4} \div \frac{2}{7} = \frac{3}{4} \times \frac{7}{2} = \frac{21}{8} = 2 \frac{5}{8}</math> </div> <div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <p>Work out</p> <math display="block">\frac{3}{4} + \frac{2}{7} = \frac{21}{28} + \frac{8}{28} = \frac{29}{28} = 1 \frac{1}{28}</math> </div> <div style="border: 1px solid black; padding: 5px;"> <p>Work out</p> <math display="block">\frac{3}{4} - \frac{2}{7} = \frac{21}{28} - \frac{8}{28} = \frac{13}{28}</math> </div>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <math display="block">2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}</math> <p>Mixed Number <span style="float: right;">Improper Fraction</span></p> </div> <p>Write <math>\frac{7}{8}</math> as a decimal.</p> $\frac{7}{8} = 8 \overline{)7.000} = 8 \overline{)7.000} = 0.875$ <p><b>Find 30% of 70. 21</b></p> <p><math>30\% = \frac{3}{10}</math> So we can find 30% by dividing by 10, then multiplying by 3. <math>70 \div 10 = 7</math>  <math>7 \times 3 = 21</math></p> <p><b>Increase 60 by 20%</b></p> $100\% = 60$ $20\% = 12$ $60 + 12 = 72$ <p><b>Decrease 80 by 45%</b></p> $100\% = 80$ $45\% = 36$ $80 - 36 = 44$

# Year 9 Term 3 Maths Knowledge Organiser [H unit 4 - Fractions, ratio & percentages] TKAT

CORE	GOOD TO KNOW...	HOW TO....
<p><b>Operations with Fractions</b></p> <ul style="list-style-type: none"> <li>- <b>Add/ subtract</b> fractions by finding equivalent fractions with the same denominator</li> <li>- <b>Multiply</b> fractions by multiplying the numerators together and the denominators together</li> <li>- <b>Divide</b> fractions by following the KFC rule: keep the first fraction as it is, flip the second fraction around so the numerator becomes the denominator and change the sign from a divide to times.</li> </ul> <p><b>Ratios</b></p> <p>A unit ratio is a ratio written in the form 1 : n, where n is a number</p> <p><b>Keywords</b></p> <p><b>Fraction</b> - A fraction represents a part of a whole.</p> <p><b>Decimal</b> - A number with a decimal point in it.</p> <p><b>Percentage</b> - A relative value indicating hundredth parts of any quantity e.g 1% of £100 = £1</p> <p><b>Numerator</b> - The part of a fraction that is above the line and signifies the number to be divided by the denominator.</p> <p><b>Denominator</b> - The part of a fraction that is below the line and that functions as the divisor of the numerator.</p> <p><b>Ratio</b> - A ratio shows how much of one thing there is compared to another.</p> <p><b>Simple interest</b> - is the interest calculated only on the original amount invested. It is the same each year.</p>	<p><b>Direct proportion</b> means that one quantity increases at the same rate as the other.</p> <ul style="list-style-type: none"> <li>- If one banana costs 20p, three bananas will cost 60p etc. The amount of bananas increase by x3 and the cost also increases by x3 so both are in direct proportion.</li> </ul> <p>There are 20 students in a class. 6 are male. What percentage of the class is male?</p> <p>Method A: <math>\frac{6}{20} \times 100\% = 6 \times \frac{100}{20} \times 1 = 30\%</math></p> <p>Method B: <math>\frac{6}{20} = \frac{30}{100} = 30\%</math> <span style="border: 1px solid red; padding: 2px;">Convert to a fraction with denominator 100.</span></p> <div style="border: 1px solid gray; border-radius: 10px; padding: 5px; margin-bottom: 5px;"> <p>Work out</p> <math display="block">\frac{3}{4} \times \frac{2}{7} = \frac{3 \times 2}{4 \times 7} = \frac{6}{28} = \frac{3}{14}</math> </div> <div style="border: 1px solid gray; border-radius: 10px; padding: 5px; margin-bottom: 5px;"> <p>Work out</p> <math display="block">\frac{3}{4} \div \frac{2}{7} = \frac{3}{4} \times \frac{7}{2} = \frac{21}{8} = 2 \frac{5}{8}</math> </div> <div style="border: 1px solid gray; border-radius: 10px; padding: 5px; margin-bottom: 5px;"> <p>Work out</p> <math display="block">\frac{3}{4} + \frac{2}{7} = \frac{21}{28} + \frac{8}{28} = \frac{29}{28} = 1 \frac{1}{28}</math> </div> <div style="border: 1px solid gray; border-radius: 10px; padding: 5px;"> <p>Work out</p> <math display="block">\frac{3}{4} - \frac{2}{7} = \frac{21}{28} - \frac{8}{28} = \frac{13}{28}</math> </div>	<div style="background-color: #fff9c4; padding: 10px; margin-bottom: 10px;"> <math display="block">2\frac{3}{4} = \frac{(4 \times 2) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}</math> <p>Mixed Number <span style="float: right;">Improper Fraction</span></p> </div> <p>Write <math>\frac{7}{8}</math> as a decimal.</p> $\frac{7}{8} = 8 \overline{)7.000} = 8 \overline{)7.000} = 0.875$ <p><math>\frac{7}{8} = 0.875</math></p> <div style="text-align: center; color: #0070c0; font-weight: bold; font-size: 1.2em;">Share \$48 in the ratio 3:1:2</div> <ol style="list-style-type: none"> <li>1) Find the <b>total number of parts</b></li> </ol> $3 + 1 + 2 = 6$ <ol style="list-style-type: none"> <li>2) Divide the <b>amount</b> by the <b>total number of parts</b></li> </ol> $\$48 \div 6 = \$8 = 1 \text{ part}$ <ol style="list-style-type: none"> <li>3) Multiply each number in the <b>ratio</b> by the value of <b>1 part</b></li> </ol> <div style="text-align: center; margin-top: 20px;">  <p style="font-size: 1.5em; font-weight: bold; color: #0070c0;">3 : 1 : 2</p> <p style="font-size: 1.5em; font-weight: bold; color: #0070c0;">x \$8      x \$8      x \$8</p> <p style="font-size: 2em; font-weight: bold; color: #0070c0;">\$24 : \$8 : \$16</p> </div>

## CORE

## GOOD TO KNOW...

## HOW TO....

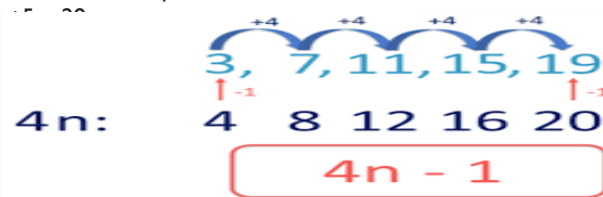
- A letter represents an unknown variable
- Manipulate an equation using inverse operations, e.g. make  $x$  the subject of the equation i.e. rearrange the equation so that  $x$  is on its own
  - $x + y = 7$  becomes  $x = 7 - y$  by subtracting  $y$  from both sides
- The letter  $n$  is generally used for sequences
- Continue a pictorial or numerical sequence - e.g. the first 4 terms in a sequence are 4, 7, 10, 13 the next term is 16 as the pattern is going up by 3 each time
- Inequalities can be written as an equation or represented on a number line
  - $<$  means less than
  - $>$  means greater than
  - $\leq$  means less than or equal to
  - $\geq$  means greater than or equal to
- Expand - multiply term outside the bracket by all terms inside the brackets to eliminate brackets
- Substitute - replace the given letter with the given value
- Solve - find the exact value of the unknown variable
- Term - is a number in a sequence e.g. 1st term etc
- Inverse - opposite, e.g. inverse of add is subtract
- Expression - Numbers, symbols and operators grouped together e.g.  $2x + 3$  is an expression
- Equation - an expression that contains an equals sign
- Identity - an equation that is true no matter what values are chosen
- Formula - a mathematical rule
- Sequence - a list of numbers or objects in a particular order
- Integer - a whole number

- Make  $x$  the subject means rearrange the equation so that  $x$  is on its own on one side
  - Use changing the subject and inverse operations to solve equations
  - When multiplying or dividing both sides of an inequality by a negative number the inequality sign reverses
  - When we solve equations, we use inverse operations to work out the value of  $x$ .
- E.g. solve  $3x + 4 = 40$
- $$\begin{array}{r} -4 \quad -4 \\ 3x = 36 \\ \div 3 \quad \div 3 \\ x = 12 \end{array}$$
- The  $n$ th term of a sequence is the general rule to work out any term in that sequence.
  - Integers solutions can be given for inequalities  
E.g. write the integer solutions which satisfy the inequality  $1 < x \leq 5$

The integer solutions would be 2, 3, 4 and 5

- To continue a sequence, we need to find the term-to-term rule

E.g. A sequence starts 3, 8, 13, 18, ... Find the next two terms in the sequence. The rule is  $+5$  so  $18 + 5 = 23$  and  $23 + 5 = 28$



$$3(a+4) = 3a + 12$$

$$2x(x+y) = 2x^2 + 2xy$$

$$4(a-5) = 4a - 20$$

Make  $c$  the subject

$$A = 3b + 3c$$

$$A - 3b = 3c$$

$$\frac{A - 3b}{3} = \frac{3c}{3}$$

$$\frac{A - 3b}{3} = \frac{3c}{3}$$

$$\frac{A - 3b}{3} = c$$

Subtract  $3b$

Divide by 3

Cancel out on the RHS

Problem:  $2x - 5 < 1$

Solution:  $2x - 5 + 5 < 1 + 5$

$$2x < 6$$

$$\frac{2x}{2} < \frac{6}{2}$$

$$x < 3$$

## Inequalities on a Number Line

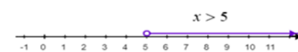
Symbol

Words

Example

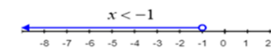
$>$

Greater than



$<$

Less than



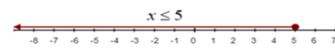
$\geq$

Greater than or equal to



$\leq$

Less than or equal to



## CORE

## GOOD TO KNOW...

## HOW TO....

- A letter represents an unknown variable
- Manipulate an equation using inverse operations, e.g. make  $x$  the subject of the equation i.e. rearrange the equation so that  $x$  is on its own
  - $x + y = 7$  becomes  $x = 7 - y$  by subtracting  $y$  from both sides
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- Equation - an expression that contains an equals sign
- Identity - an equation that is true no matter what values are chosen
- Formula - a mathematical rule
- Sequence - a list of numbers or objects in a particular order
- Integer - a whole number

- Make  $x$  the subject means rearrange the equation so that  $x$  is on its own on one side
- Use changing the subject and inverse operations to solve equations
- When multiplying or dividing both sides of an inequality by a negative number the inequality sign reverses
- When we solve equations, we use inverse operations to work out the value of  $x$ .

E.g. solve  $3x + 4 = 40$

$$\begin{array}{r} -4 \quad -4 \\ 3x = 36 \\ \div 3 \quad \div 3 \\ x = 12 \end{array}$$

- The  $n$ th term of a sequence is the general rule to work out any term in that sequence.
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E.g. A sequence starts 3, 8, 13, 18, ... Find the next two terms in the sequence. The rule is  $+5$  so  $18 + 5 = 23$  and  $23 + 5 = 28$

4n: 4 8 12 16 20

**4n - 1**

$$3(a+4) = 3a + 12 \quad 2x(x+y) = 2x^2 + 2xy$$

$$4(a-5) = 4a - 20 = 2x^2 + 2xy$$

Make  $c$  the subject

$$A = 3b + 3c$$

$$A - 3b = 3c$$

$$\frac{A - 3b}{3} = \frac{3c}{3}$$

$$\frac{A - 3b}{3} = \frac{3c}{3}$$

$$\frac{A - 3b}{3} = c$$

Subtract  $3b$

Divide by 3

Cancel out on the RHS

Problem:  $2x - 5 < 1$

Solution:  $2x - 5 + 5 < 1 + 5$

$$2x < 6$$

$$\frac{2x}{2} < \frac{6}{2}$$

$$x < 3$$

$$x < 3$$

## Inequalities on a Number Line

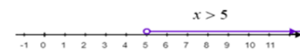
Symbol

Words

Example

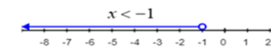
$>$

Greater than



$<$

Less than



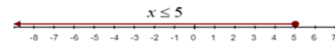
$\geq$

Greater than or equal to



$\leq$

Less than or equal to



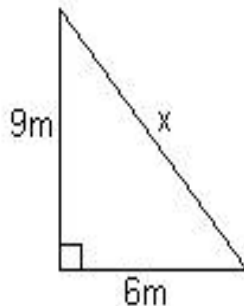


## CORE

## GOOD TO KNOW...

## HOW TO....

- Angles are measured in degrees
- Angles on a straight line add up to 180
- A right angle is 90 degrees
- Angles around a point add up to 360
- Angles in a triangle add up to 180
- Angles in a quadrilateral add up to 360
- Exterior angles add up to 360
- Vertically opposite angles are equal
- Co-interior angles add up to 180
- Alternate angles are equal
- Corresponding angles are equal



$$x^2 = 6^2 + 9^2$$

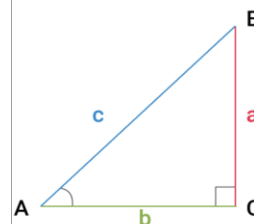
$$x^2 = 36 + 81$$

$$x^2 = 117$$

$$x = \sqrt{117}$$

$$x = 10.8\text{m (1dp)}$$

### SOH - CAH - TOA



$$\text{sine of } \angle A = \sin A = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{a}{c}$$

$$\text{cosine of } \angle A = \cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{b}{c}$$

$$\text{tangent of } \angle A = \tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{a}{b}$$

Calculate  $p$ . Give your answer correct to 2 decimal places.

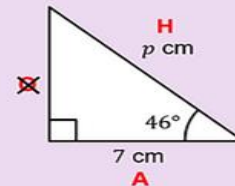
$$\cos \theta = \frac{A}{H}$$

$$\cos 46 = \frac{7}{p}$$

$$p \times \cos 46 = 7$$

$$p = \frac{7}{\cos 46}$$

$$p = 10.08 \text{ cm}$$



### Exterior Angles

The sum of the exterior angles of any polygon is  $360^\circ$ .

The exterior angle of a regular  $n$ -sided polygon is  $\frac{360^\circ}{n}$

### Formula for the sum of Polygon interior angles

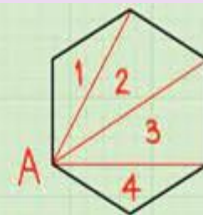
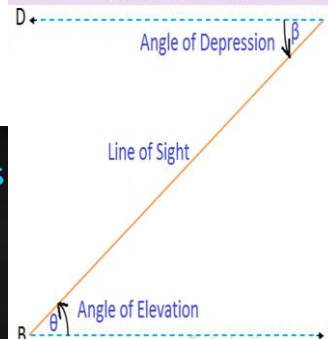
square

4

$$\text{sum} = (n - 2) \times 180^\circ$$

$$= (4 - 2) \times 180^\circ$$

$$= 360^\circ$$



$$4 \times 180^\circ = 720^\circ$$



## CORE

## GOOD TO KNOW...

## HOW TO....

### Geometry

## Geometry Basics

### Corresponding and Alternate Angles

### GCSE Maths

MME  
mathsmadeeasy.co.uk

- Angles are measured in degrees
- Angles on a straight line add up to 180
- A right angle is 90 degrees
- Angles around a point add up to 360
- Angles in a triangle add up to 180
- Angles in a quadrilateral add up to 360
- Exterior angles add up to 360
- Vertically opposite angles are equal
- Co-interior angles add up to 180
- Alternate angles are equal
- Corresponding angles are equal

- Names of 2D shapes
  - 5 sided - Pentagon
  - 6 sided - Hexagon
  - 7 sided - Heptagon
  - 8 sided - Octagon
  - 9 sided - Nonagon
  - 10 sided - Decagon

- $A <$  is used to label an angle.
- $\angle ABC$  refers to the middle letters so we would be looking at the angle at B shown below.

The figure below also illustrates the angle  $\angle ABC$

The outside Angle of a Triangle always equals the sum of the two far away inside angles.

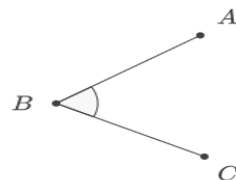
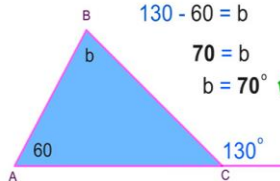
$$\text{Exterior Angle } C^\circ = a + b$$

$$130 = 60 + b$$

$$130 - 60 = b$$

$$70 = b$$

$$b = 70^\circ \checkmark$$



- Parallel - Two straight lines equidistant apart which never meet

- Polygon - a 2D shape made up of straight lines
- Equilateral triangle - a triangle with 3 equal angles and sides
- Isosceles triangle - a triangle with 2 equal angles and sides
- Right angled triangle - a triangle with one right angle
- Interior angle - an angle inside a polygon
- Exterior - an angle outside a polygon
- Perpendicular - at 90 degrees to a given line

- Congruent - a shape that is exactly the same shape and size
- Similar - a shape with the same size angles with all corresponding sides in proportion

## Formula for the sum of Polygon interior angles

square

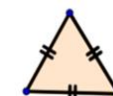
4

$$\begin{aligned} \text{sum} &= (n - 2) \times 180^\circ \\ &= (4 - 2) \times 180^\circ \\ &= 360^\circ \end{aligned}$$

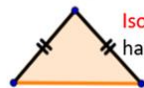
## Exterior Angles

The sum of the exterior angles of any polygon is  $360^\circ$ .

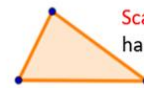
The exterior angle of a regular n-sided polygon is  $\frac{360^\circ}{n}$



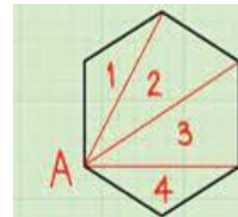
**Equilateral Triangle**  
has three equal sides



**Isosceles Triangle**  
has two equal sides



**Scalene Triangle**  
has no equal sides



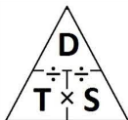
$$4 \times 180^\circ = 720^\circ$$

## CORE

## GOOD TO KNOW...

## HOW TO....

- A linear graph is a straight line
- Quadratic, cubic and reciprocal graphs are curved
- $y = mx + c$  represents a linear graph where  $m$  is the gradient and  $c$  is the  $y$  intercept
- The mid point is halfway between the two given points
- The diameter is double the radius



## Keywords

- Linear - when graphed creates a straight line
- Quadratic - one unknown term is squared
- Cubic - one unknown term is cubed
- Equation - an expression that contains an equals sign
- Root - a solution to a quadratic or cubic equation. There can be more than one root
- Origin - the point where the  $x$  and  $y$  axes intersect
- Axis - the horizontal or vertical number line which intersect to create a coordinate grid
- Gradient - the steepness of a line
- $Y$ -intercept - the point where a line cuts the  $y$  axis
- Proportion - a mathematical comparison between two numbers - if the ratios that the two numbers increase/decrease are the same this is direct proportion
- Perpendicular - at  $90$  degrees to a given line

- Know that a graph axis doesn't have to start at zero but can start at any number using a zigzag between the origin and the first defined number
- Be able to find the equation of a line perpendicular to a given line
- Use the formula to calculate the gradient of a graph

$$m = \frac{\text{difference in } y}{\text{difference in } x}$$

- Substitute values into an equation to formulate a table of values to create a graph
- Understand and interpret distance time graphs, velocity graphs and calculate rates of change
- Calculate area under graph
- Know that a quadratic and cubic equation can have more than 1 solution
- Acceleration =  $\frac{\text{change in velocity}}{\text{time}}$
- The equation for a circle with centre  $(0, 0)$ , is given by the equation  $x^2 + y^2 = r^2$  where  $r$  is the radius

Find the equation of the line that is perpendicular to

$$y = -\frac{1}{3}x + 4 \text{ and passes through } (9, -5)$$

$$m = 3$$

$$y = 3x + b$$

$$-5 = 3(9) + b$$

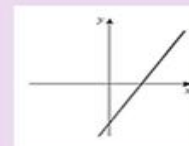
$$-5 = 27 + b$$

$$-27 - 27$$

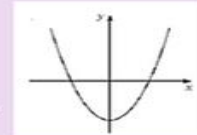
$$-32 = b$$

$$y = 3x - 32$$

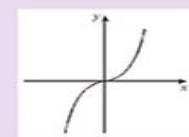
## Types of Graphs



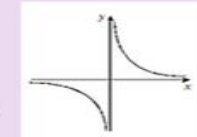
Linear



Quadratic



Cubic



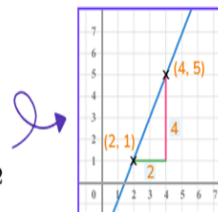
Reciprocal

To calculate the gradient of a straight line through two coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$ :

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

E.g.

$$m = \frac{5 - 1}{4 - 2} = \frac{4}{2} = 2$$



It can be helpful to think about this formula as: "Change in  $y$  divided by change in  $x$ " or "Rise over run"

## CORE

## GOOD TO KNOW...

## HOW TO....

- When we order a set of numbers, we need to line them up either:
  - Ascending - smallest value to biggest
  - Descending - biggest to smallest
- Sampling - using a portion of a total population to represent the full population
- Mean - an average calculated by adding all the values and dividing by the total number of values
- Mode - the most common value
- Median - list numbers in numerical order and find the middle value
- Range - the biggest value minus the smallest value
- Outlier - a data point which doesn't fit the trend of the rest of the data

Example: Parking Spaces per House in Hampton Street

Isabella went up and down the street to find out how many parking spaces each house has. Here are her results:

Parking Spaces	Frequency
1	15
2	27
3	8
4	5

What is the mean number of Parking Spaces?

Answer:

$$\begin{aligned} \text{Mean} &= \frac{15 \times 1 + 27 \times 2 + 8 \times 3 + 5 \times 4}{15 + 27 + 8 + 5} \\ &= \frac{15 + 54 + 24 + 20}{55} \\ &= 2.05... \end{aligned}$$

The Mean is **2.05** (to 2 decimal places)

Example:

The following is a frequency table of the score obtained in a mathematics quiz. Find the median score.

Score	0	1	2	3	4
Frequency	3	4	7	6	3

Solution:

Number of scores = 3 + 4 + 7 + 6 + 3 = 23 (odd number)

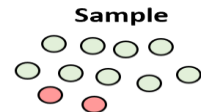
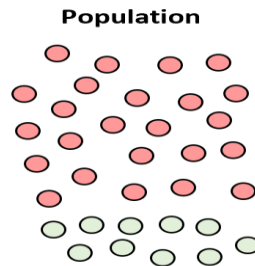
Since the number of scores is odd, the median is at the  $\left(\frac{n+1}{2}\right)^{\text{th}} = \left(\frac{23+1}{2}\right)^{\text{th}} = 12^{\text{th}}$  position.

To find out the 12<sup>th</sup> position, we need to add up the frequencies as shown:

Score	0	1	2	3	4
Frequency	3	4	7	6	3
Position	3	3 + 4 = 7	7 + 7 = 14		

The 12<sup>th</sup> position is after the 7<sup>th</sup> position but before the 14<sup>th</sup> position. So, the median is 2.

Marks scored	Frequency	Mid-point	Frequency × Mid-point
0 - 9	3	$\frac{0+9}{2} = 4.5$	$3 \times 4.5 = 13.5$
10 - 19	5	$\frac{10+19}{2} = 14.5$	$5 \times 14.5 = 72.5$
20 - 29	8	$\frac{20+29}{2} = 24.5$	$8 \times 24.5 = 196$
30 - 39	4	$\frac{30+39}{2} = 34.5$	$4 \times 34.5 = 138$
	<b>n = 20</b>		<b>Total = 420</b>



6 is recorded as 06

The key shows us how to read the diagram

KEY: 2 | 5 means 25

0	6	7	8						
1	0	2	3	4	7	7	7	8	9
2	1	3	4	5	7				
3	1	1	2	6	6	9			
4	1	5	5	6	9				
5	0	0							

This number is 39



Goals Scored Over the Last 7 Games

1 3 4 6 6 7 8

mean average **5**

mode most common **6**

median middle **6**

range largest - smallest **7**

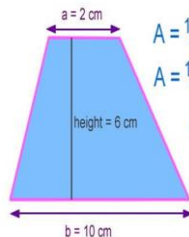
## CORE

## GOOD TO KNOW...

## HOW TO....

- 2D - a 2 dimension shape (flat, e.g. square, circle)
- 3D - a 3 dimensional shape (solid, e.g. cube, cylinder)
- Area - the space inside a 2D shape measured in squared units
- Area of a rectangle/square = length x width
- Area of a triangle =  $\frac{\text{base} \times \text{height (perpendicular)}}{2}$
- Area of a circle =  $\pi r^2$
- Circumference of a circle =  $\pi d$
- Area of a trapezium =  $\frac{(a+b)h}{2}$
- Volume - the space inside a 3D shape measured in cubed units
- Volume of a cube/cuboid = length x width x height
- Volume of a prism = area of cross section (front face) x length
- Volume of a cylinder =  $\pi r^2 \times h$
- Know properties of triangles and quadrilaterals
- Prism - a 3D shape which has the same cross section throughout, e.g. cuboid, cylinder, triangular prism)
- Arc - a curve joining two points on the circumference of a circle
- Sector - a region of a circle bounded by two radii and an arc

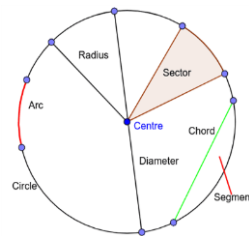
- Plan is the view of a 3D shape when looked at from above
- Elevation is the view of a 3D shape when looked at from the front or the side
- Be able to find the area of a compound shape
- Understand and use bounds
- Surface area is the area of all the faces of a 2D shape added together
- Perpendicular - at an angle of 90 degrees to a given line
- Know that a hemisphere is half a sphere
- Recognise and identify cones, pyramids and frustums
- Identify arcs, sectors and segments



$$A = \frac{1}{2}(a+b) \times h$$

$$A = \frac{1}{2}(2+10) \times 6$$

$$A = 36 \text{ cm}^2$$



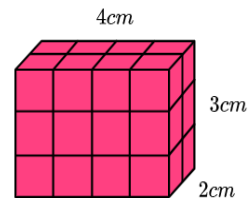
1m



100cm



1 metre squared = 10,000 centimetre squared



This cuboid is made from 24 unit cubes.

Its volume is

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$

$$\text{Volume} = 2 \times 4 \times 3$$

$$\text{Volume} = 24 \text{ cm}^3$$

What is the area of a circle with radius 3cm?



$$\text{Area} = \pi r^2$$

$$= \pi \times 3^2$$

$$= 9\pi \text{ cm}^2$$

$$= 28.3 \text{ cm}^2 \text{ (1.d.p.)}$$

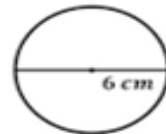
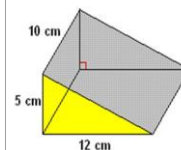
$$V = \frac{(b \times h) \times H}{2}$$

$$V = \frac{(12 \times 5) \times 10}{2}$$

$$V = \frac{60 \times 10}{2}$$

$$V = 30 \times 10$$

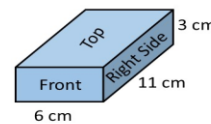
$$V = 300 \text{ cm}^3$$



$$C = \pi d$$

$$= 3.142 \times 6 \text{ cm}$$

$$= 18.85 \text{ cm}$$



Front  $6 \times 3 = 18$

Front & Back  
(2 × 18) = 36

Right Side  $11 \times 3 = 33$

Right & Left Side  
(2 × 33) = 66

Top  $6 \times 11 = 66$

Top & Bottom  
(2 × 66) = 132

Total Surface Area = 234 cm<sup>2</sup>

## CORE

## GOOD TO KNOW...

## HOW TO....

### Perimeter

- Calculated by adding up the length of each of the sides.

### Area

- Square/Rectangle = length x width
- Triangle =  $\frac{1}{2} \times \text{base} \times \text{height}$
- Measure in squared units, e.g.  $\text{cm}^2$ .

### Volume

- Cube/cuboid = length x width x height
- Prism = area of cross section (front face) x length
- Measure in cubic units, e.g.  $\text{cm}^3$ .

### Keywords

Perimeter - The distance around the edge of a shape.

Area - The space inside a 2D shape.

Volume - The volume of a 3D shape is the amount of space inside it.

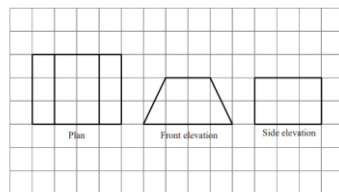
Surface Area - The amount of space covering the outside of a 3D shape.

Perpendicular Height - The line at a right angle to the base line.

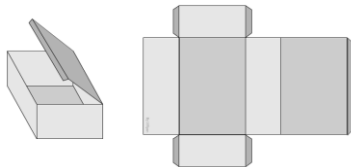
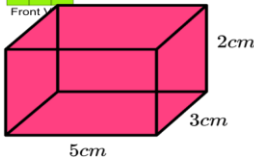
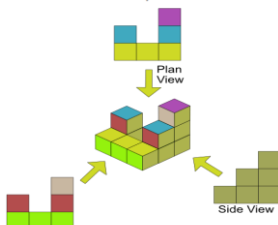
Prism - A 3D shape which has the same cross section throughout, (e.g. cuboid, cylinder, triangular prism).

Parallelogram - A quadrilateral with two pairs of parallel sides. Looks like a slanted rectangle.

The diagram shows the plan, front elevation and side elevation of a solid shape, drawn on a centimetre grid.



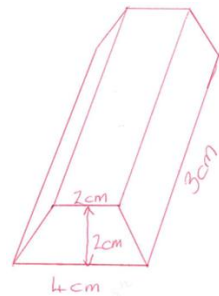
In the space below, draw a sketch of the solid shape. Give the dimensions of the solid on your sketch.



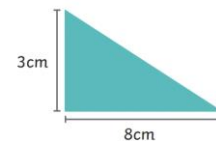
Face	Area
Bottom	$5 \times 3 = 15$
Top	15
Front	$5 \times 2 = 10$
Back	10
Right side	$2 \times 3 = 6$
Left side	6

$$\text{Total surface area} = 15 + 15 + 10 + 10 + 6 + 6 = 62\text{cm}^2$$

$$\begin{aligned} \text{Volume of cuboid} &= \text{length} \times \text{width} \times \text{height} \\ &= 5 \times 3 \times 2 \\ &= 30\text{cm}^3 \end{aligned}$$



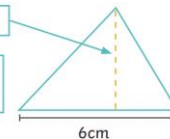
$\text{base} \times \text{perpendicular height} \div 2 = \text{area of a triangle}$



$$8\text{cm} \times 3\text{cm} \div 2 = 12\text{cm}^2$$

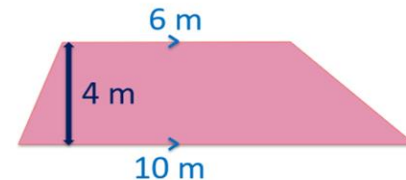
**perpendicular height** = 5cm

$$6\text{cm} \times 5\text{cm} \div 2 = 15\text{cm}^2$$



To find the area of a trapezium, add the parallel sides, divide by 2

then multiply by the distance between the parallel sides



$$\text{Area} = \left(\frac{a+b}{2}\right)h = \left(\frac{6+10}{2}\right) \times 4 = 8 \times 4 = 32\text{m}^2$$



## CORE

## GOOD TO KNOW...

## HOW TO....

- Transformations - Transformations change the size or position of shapes. There are four types of transformations: reflections, enlargements, rotations, translations.
- Reflection - A shape can be reflected across a line of reflection to create an image, like looking in a mirror. The line of reflection is also called the mirror line. Every point in the image is the same distance from the mirror line as the original shape.
- Rotation - Rotation turns a shape around a fixed point called the centre of rotation. There are three things needed to rotate a shape: the centre of rotation (a coordinate), the angle of rotation ( $90^\circ$ ,  $180^\circ$  etc.) and the direction of rotation (clockwise or anti-clockwise)
- Translation - A translation moves a shape up, down or from side to side but it does not change its appearance in any other way.
- Enlargement - Enlarging a shape changes its size. The shape can get either bigger or smaller. Two things are needed to enlarge a shape: scale factor (x2 would make a side twice as big) and the centre of enlargement (a coordinate)
- 3D shapes can be drawn from different viewpoints.
- The plan looks at a shape from above (the birdseye view)
- The front elevation looks at a shape from the front
- The side elevation looks at a shape from the side
- We draw the plan and elevations as 2D shapes.

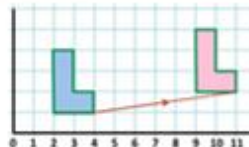
# TRANSFORMATIONS

A CHANGE IN THE POSITION OR SIZE OF AN OBJECT

### TRANSLATION

Described by a vector

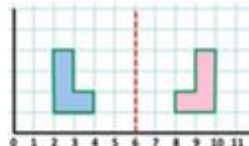
VECTOR  $\begin{pmatrix} 7 \\ 1 \end{pmatrix}$



### REFLECTION

Described by a mirror line

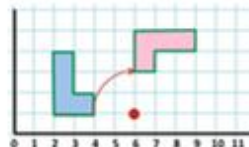
MIRROR LINE  $x = 6$



### ROTATION

Described by an angle and a centre

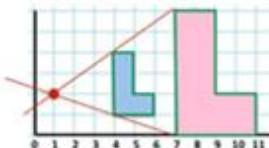
$90^\circ$  CLOCKWISE, CENTRE (6, 1)



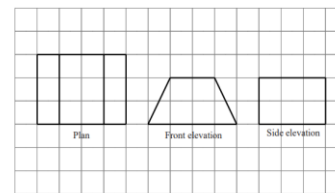
### ENLARGEMENT

Described by a scale factor and a centre

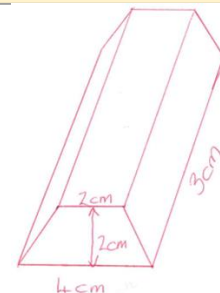
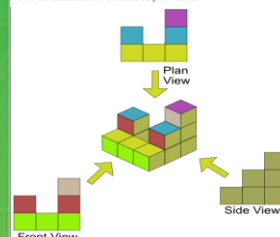
SCALE FACTOR = 2, CENTRE (1, 2)



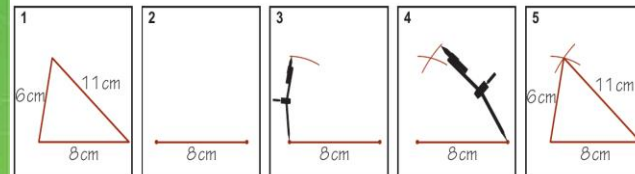
The diagram shows the plan, front elevation and side elevation of a solid shape, drawn on a centimetre grid.



In the space below, draw a sketch of the solid shape. Give the dimensions of the solid on your sketch.



Construct a triangle with sides 11 cm, 8 cm and 6 cm.



- 1 Sketch the triangle first.
- 2 Draw the 8 cm line.
- 3 Open your compasses to 6 cm. Place the point at one end of the 8 cm line. Draw an arc.
- 4 Open your compasses to 11 cm. Draw another arc from the other end of the 8 cm line. Make sure your arcs are long enough to intersect.
- 5 Join the intersection of the arcs to each end of the 8 cm line. Don't rub out your construction marks.





# Knowledge Organisers

## Year 10

## CORE

**Linear graphs** are straight line graphs, e.g.  $y = 2x - 1$ .

- We substitute the x value into the equation to get the y value. Once we have both we can then plot the coordinates and draw the graph.

**Y-axis:** the vertical axis on a grid

**X-axis:** the horizontal axis on a grid

**Midpoint:** The middle value of a coordinate. To find the midpoint, add the two x-values together and half them, then add the two y-values together and half them.

### Keywords

**Coordinates:** A pair of numbers that describe the position of a point on a graph with x and y axis.

**Line Segment:** A part of a straight line that is bounded by two distinct endpoints.

**Gradient:** How steep a line is at any point.

**Midpoint:** The point halfway along a line or between two coordinates.

**Intercept:** Where two graphs cross.

**y-intercept:** Where a graph crosses the y-axis.

**Parallel:** Two straight lines that stay the same distance apart.

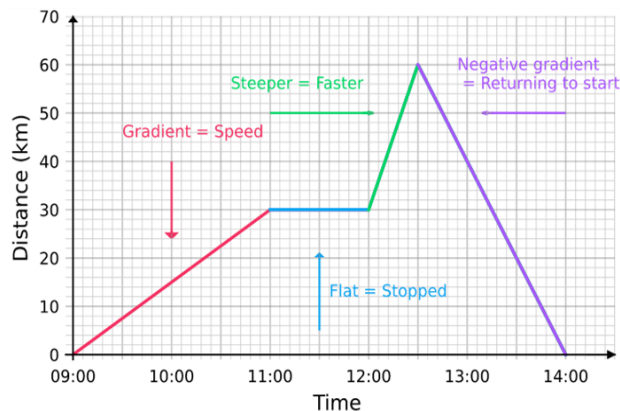
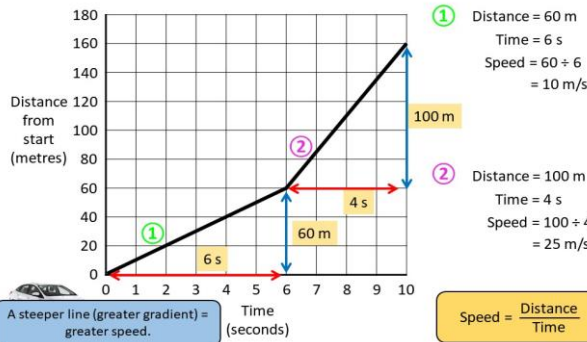
positive gradient

negative gradient

## GOOD TO KNOW...

A car's distance is recorded for 10 seconds.

How can we calculate the 2 different speeds?



## HOW TO....

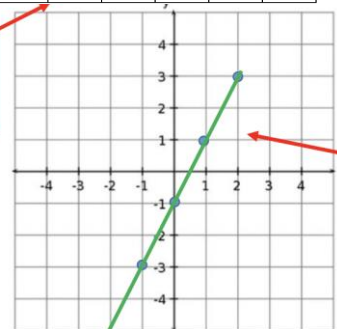
Draw the linear graph of  $y = 2x - 1$ .

$$y = 2x - 1$$

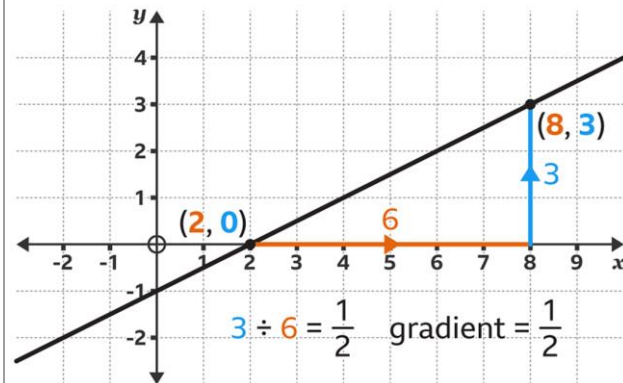
x	-2	-1	0	1	2
y	-5	-3	-1	1	3

Multiply this value by 2 and then subtract 1 to get the y value.

This coordinate would be (-2,-5).



Don't forget to draw a straight line through all of the coordinates you have plotted.



## CORE

## GOOD TO KNOW...

## HOW TO....

# FIND THE ROOTS OF

$$f(x) = x^2 + 18x + 65$$

QUADRATIC FORMULA	FACTORING	COMPLETING THE SQUARE
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $f(x) = 1x^2 + 18x + 65$ $\frac{-18 \pm \sqrt{18^2 - 4(1)(65)}}{2(1)}$ $\frac{-18 \pm \sqrt{64}}{2}$ $\frac{-18 + 8}{2} \quad \frac{-18 - 8}{2}$ $x = -5 \quad x = -13$	$0 = x^2 + 18x + 65$ <p>What two numbers have a sum of 18 and a product of 65?</p> <p>5 and 13</p> $0 = (x + 5)(x + 13)$ $x = -5 \quad x = -13$	$0 = x^2 + 18x + 65$ $-65 = x^2 + 18x$ <p>Take half of 18, square it, and add to both sides</p> $9^2 - 65 = x^2 + 18x + 9^2$ $16 = (x + 9)^2$ $\sqrt{16} = \sqrt{(x + 9)^2}$ $\pm 4 = x + 9$ $x = -5 \quad x = -13$

## Simultaneous Equations

By elimination method

1. Two linear equations

**Example**

Solve (i)

$$2x + y = 5$$

$$x - y = 1$$

$$3x = 6$$

$$x = 2$$

$$x - y = 1$$

$$2 - y = 1$$

$$y = 1$$

$$y = 1$$

$$y = 1$$

$$y = 1$$

$$y = 1$$

Eliminate y by adding the equations

Putting value back into 2<sup>nd</sup> equation

When the signs of the **equal terms** are **DIFFERENT**, we **ADD** together the two equations to eliminate x.

You can write the solution to an inequality using **set notation**.

$$\{x : x > 2\}$$

the set of  $x$  such that

Solve  $3x - 2 > 6 - x$ . Show your answer on a number line and write the solution set using set notation.

$$3x > 6 - x + 2 \quad \text{Add 2 to both sides.}$$

$$4x > 8 \quad \text{Add } x \text{ to both sides.}$$

$$x > 2 \quad \text{Divide both sides by 4.}$$

In set notation:  $\{x : x > 2\}$  — This tells us that there is a set of values of  $x$ , not just one value.

$$\text{Solution: } 2x - 5 + 5 < 1 + 5$$

$$2x < 6$$

$$2x < 6$$

$$\frac{2x}{2} < \frac{6}{2}$$

$$x < 3$$

## Inequalities on a Number Line

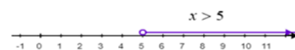
Symbol

Words

Example

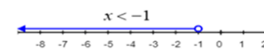
>

Greater than



<

Less than



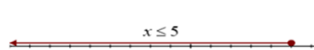
≥

Greater than or equal to



≤

Less than or equal to



- Solving a quadratic equation means finding values for the unknown that fit.
- You can solve quadratics by either: factorising, completing the square or by using the quadratic formula. All give you the exact same answer.
- The roots of a quadratic function are its solutions when it is equal to zero.
- When there are two unknowns, you need two equations to find their values. These are called simultaneous equations.

## Inequalities

- You can show inequalities on a number line.
  - An empty circle shows the value is not included.
  - A filled circle shows the value is included.
  - An arrow shows the solution continues towards infinity.
  - < means less than
  - > means greater than
  - ≤ means less than or equal to
  - ≥ means greater than or equal to
- You can rearrange an inequality in the same way as you rearrange an equation.

## CORE

## GOOD TO KNOW...

## HOW TO....

### Transformations -

Transformations change the size or position of shapes. There are four types of transformations:

**1. Reflection** - A shape can be reflected across a line of reflection to create an image, like looking in a mirror.

- The line of reflection is also called the mirror line. Every point in the image is the same distance from the mirror line as the original shape.

**2. Rotation** - Rotation turns a shape around a fixed point called the centre of rotation.

- There are three things needed to rotate a shape: the centre of rotation (a coordinate), the angle of rotation ( $90^\circ$ ,  $180^\circ$  etc.) and the direction of rotation (clockwise or anti-clockwise)

**3. Translation** - A translation moves a shape up, down or from side to side but it does not change its appearance in any other way.

- **Vector** - A vector is a quantity that has both a magnitude and a direction

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$  means 3 right, 2 up      $\begin{pmatrix} -4 \\ -5 \end{pmatrix}$  means 4 left, 5 down.

**4. Enlargement** - Enlarging a shape changes its size. The shape can get either bigger or smaller.

- Two things are needed to enlarge a shape: scale factor ( $\times 2$  would make a side twice as big) and the centre of enlargement (a coordinate)

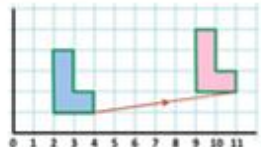
# TRANSFORMATIONS

A CHANGE IN THE POSITION OR SIZE OF AN OBJECT

### TRANSLATION

Described by a vector

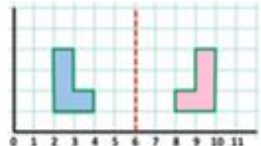
VECTOR  $\begin{pmatrix} 7 \\ 1 \end{pmatrix}$



### REFLECTION

Described by a mirror line

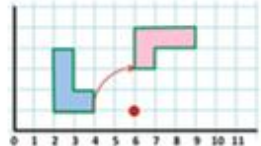
MIRROR LINE  $x = 6$



### ROTATION

Described by an angle and a centre

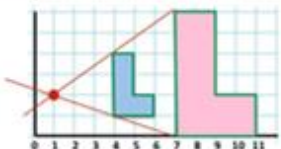
$90^\circ$  CLOCKWISE, CENTRE  $(6, 1)$



### ENLARGEMENT

Described by a scale factor and a centre

SCALE FACTOR = 2, CENTRE  $(1, 2)$

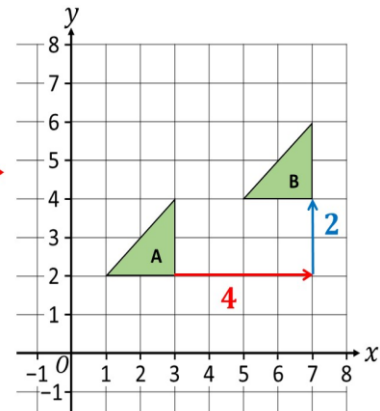


Transformations: Translation

Describe the translation of A to B with a vector.

$\begin{pmatrix} \text{Horizontal} \\ \text{Vertical} \end{pmatrix}$

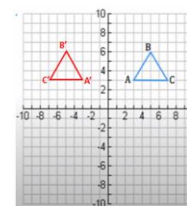
$\begin{pmatrix} 4 \\ 2 \end{pmatrix}$



(Pick a corner to measure from.)

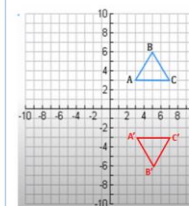
### Reflection on a Coordinate Plane

Reflect across the **y-axis**:  
 $(x,y) \rightarrow (-x,y)$



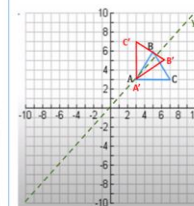
A =  $(3, 3) \rightarrow (-3, 3)$   
B =  $(5, 6) \rightarrow (-5, 6)$   
C =  $(7, 3) \rightarrow (-7, 3)$

Reflect across the **x-axis**:  
 $(x,y) \rightarrow (x,-y)$



A =  $(3, 3) \rightarrow (3, -3)$   
B =  $(5, 6) \rightarrow (5, -6)$   
C =  $(7, 3) \rightarrow (7, -3)$

Reflect across the line **y = x**:  
 $(x,y) \rightarrow (y,x)$



A =  $(3, 3) \rightarrow (3, 3)$   
B =  $(5, 6) \rightarrow (6, 5)$   
C =  $(7, 3) \rightarrow (3, 7)$

## CORE

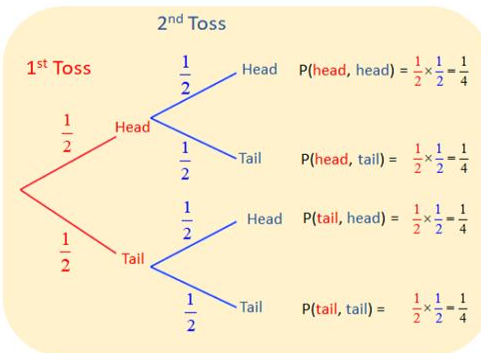
## GOOD TO KNOW...

## HOW TO....

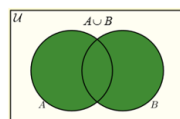
- Probability - how likely something is to happen
- Probability is either given as a fraction, decimal or a percentage
- Probability =  $\frac{\text{number of successful outcomes}}{\text{Total number of possible outcomes}}$
- A sample space diagram shows all the possible outcomes of two events.
- Two events are mutually exclusive if they cannot happen at the same time. For example, you cannot be at school and at the cinema at the same time.
- When events are mutually exclusive, you can add their probabilities together.
- Probability adds up to 1
- The probability of an event not happening, is one minus the probability of the event happening.
- In a probability experiment a trial is repeated many times and the outcomes recorded. The relative frequency of an outcome is called the experimental probability.
- Theoretical probability is calculated without doing an experiment.
- A frequency tree shows two or more events and the number of times they occurred.
- Two events are independent if one event does not affect the probability of the other. To find the probability of independent event, multiply the two probabilities together.
- A tree diagram shows two or more events and their probabilities.
- If one event depends on the other, the two events are dependent.

### Tree Diagram

Tree diagram to show the probabilities when a coin is tossed twice.

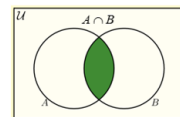


### Venn Diagrams



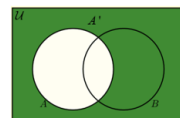
#### A union B

Elements that belong to either A or B or both.



#### A intersect B

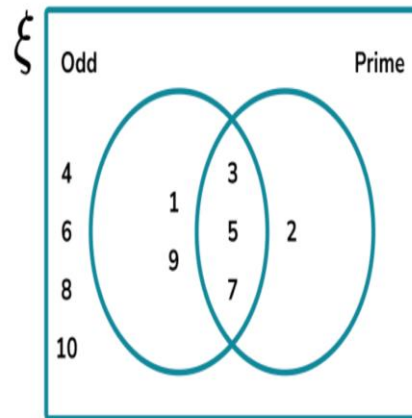
Elements that belong to both A and B.



#### A complement

Elements that don't belong to A.

Below is a Venn diagram describing the sets of odd numbers and prime numbers for the integer values in the universal set  $\xi = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ .



There are 5 odd numbers. 3 of these odd numbers are also prime numbers. The probability of selecting a number from the universal set that is odd, and prime, is 3 out of the total number of values in the universal set, 10. The solution is therefore:

$$P(\text{Odd and Prime}) = P(O \cap P) = \frac{3}{10}$$



## CORE

## GOOD TO KNOW...

## HOW TO....

### Ratios

- A ratio shows how much of one thing there is compared to another.
- A unit ratio is a ratio written in the form 1 : n, where n is a number.

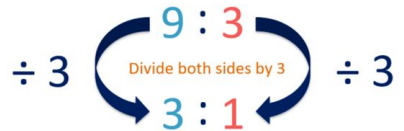
**Direct proportion** means that one quantity increases at the same rate as the other.

- If one banana costs 20p, three bananas will cost 60p etc. The amount of bananas increase by x3 and the cost also increases by x3 so both are in direct proportion.

**Inverse proportion** means that when one quantity increases, the other decreases.

- If it takes 2 decorators 6 hours to paint a room, it will take 4 decorators 3 hours to paint the same room. The number of decorators has doubled and the amount of time has halved.

To simplify a ratio, divide all numbers in the ratio by the same amount



**DEMO** Which deal is the best value?

**Eat Fresh**

3 for £2.40

$£2.30 \div 3 = 80p$  each

**CHEAPEST**

**Max-Mart**

4 for £3.40

$£3.40 \div 4 = 85p$  each

**Best Buys**

**YOUR TURN** Which deal is the best value?

**Eat Fresh**

3 for £2.70

$£2.70 \div 3 = 90p$  each

**CHEAPEST**

**Max-Mart**

2 for £1.60

$£1.60 \div 2 = 80p$  each

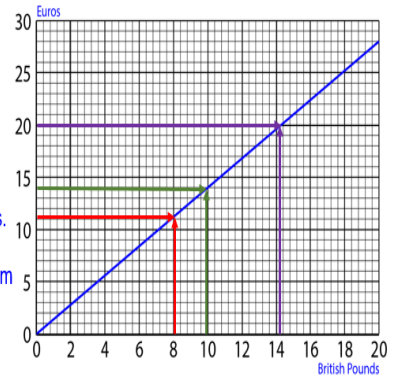
price per item = total cost ÷ quantity

**Conversion Graphs**

**Learning Objective:** Plot and interpret real life conversion graphs.

Use the graph to:

- ANS** a) convert £8 to Euros  
11€
- ANS** b) convert 14 Euros to £s  
£10
- ANS** c) find the difference in £s between £12 and 14 Euros.  
 $£12 - £10 = £2$
- ANS** d) calculate the change from £50 when you spend 20€.  
Give your answer in £s  
 $£50 - £14 = £36$



Share \$20 in the ratio 1:3

- 1) Find the **total number of parts**  
 $1 + 3 = 4$
- 2) Divide the **amount** by the **total number of parts**  
 $£20 \div 4 = £5 = 1 \text{ part}$
- 3) Multiply each number in the **ratio** by the value of **1 part**



Here are the ingredients needed to make 16 gingerbread men.

$24 - 16 = 8$

Ingredients to make 16 gingerbread men	
180 g	flour
40 g	ginger
110 g	butter
30 g	sugar



8 people  
40g  
20g  
55g  
15g

Hamish wants to make 24 gingerbread men.

Work out how much of each of the ingredients he needs.

$180 + 90 = 270$   
 $40 + 20 = 60$   
 $110 + 55 = 165$   
 $30 + 15 = 45$

270	g flour
60	g ginger
165	g butter
45	g sugar



## CORE

## GOOD TO KNOW...

## HOW TO....

- Compound interest - The amount of interest earned over a certain period of time i.e. monthly, quarterly or yearly. The interest earned is based on the current amount.
- Per annum means per year
- Compound measures combine measures of two different quantities
  - Speed is a measure of distance travelled and time taken
  - Density is the mass of a substance in a certain volume
  - Pressure is a measure of force applied over an area

**Direct proportion** means that one quantity increases at the same rate as the other.

- If one banana costs 20p, three bananas will cost 60p etc. The amount of bananas increase by x3 and the cost also increases by x3 so both are in direct proportion.

**Inverse proportion** means that when one quantity increases, the other decreases.

- If it takes 2 decorators 6 hours to paint a room, it will take 4 decorators 3 hours to paint the same room. The number of decorators has doubled and the amount of time has halved.

You borrow the money for a \$6500 car. The bank loans you the money at **7.25%** compounded **annually** and would like you to pay off the car in **5 years**. How much is your total payoff?  $A = P(1 + r)^t$

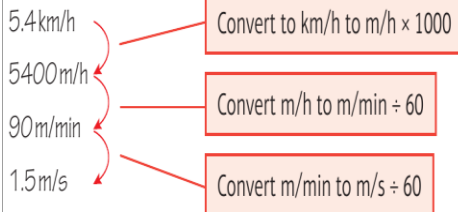
$$A = 6500(1 + 0.0725)^5$$

$$A = 6500(1.419)$$

$$A = \$9223.59$$

$7\% = 0.07$   
 $7.25\%$   
 $= 0.0725$

A man walks at an average speed of 5.4 km/h. What is his average speed in m/s?



Compound measures	
<b>Speed</b> $\text{speed} = \frac{\text{distance}}{\text{time}}$	
<b>Density</b> $\text{density} = \frac{\text{mass}}{\text{volume}}$	
<b>Pressure</b> $\text{pressure} = \frac{\text{force}}{\text{area}}$	

### Calculating Average Speed

$$\text{Total Distance Travelled} = 240 + 360 = 600 \text{ km}$$

$$\text{Total Time Taken} = 3 \text{ hour} + 4 \text{ hours} = 7 \text{ hours}$$

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

$$= \frac{600}{7}$$

$$= 85.71 \text{ km/hr}$$

Example 1: A plastic shape of density 1.08g/cm<sup>3</sup> has a volume of 225cm<sup>3</sup>. Calculate the mass of the shape.

$$\text{Mass} = \text{Density} \times \text{Volume}$$

Put numbers into formula

$$\text{Mass} = 1.08 \times 225$$

Work out 1.08 x 225

$$\text{Mass} = 243 \text{ g}$$

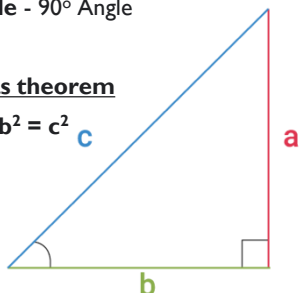
## CORE

## GOOD TO KNOW...

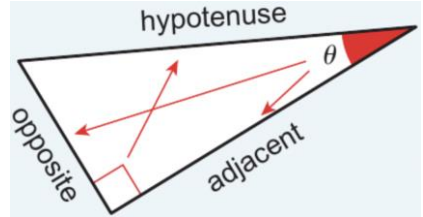
## HOW TO....

**Right Angle** -  $90^\circ$  Angle

**Pythagoras theorem**  
 $a^2 + b^2 = c^2$



**Hypotenuse:** The side opposite the right angle.  
**Opposite:** The side opposite the angle  $\theta$ .  
**Adjacent:** The side next to angle  $\theta$ .



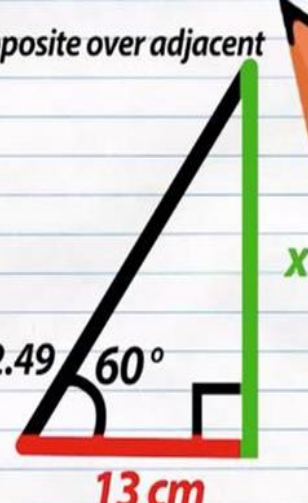
SOHCAHTOA is an easy way to remember the three main trigonometric ratios. They are Sine (Sin), Cosine (Cos) and Tangent (Tan).

**TOA** tangent is opposite over adjacent

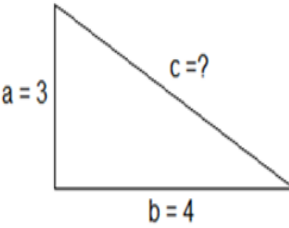
$$\tan 60 = \frac{x}{13}$$

$$\tan 60 = 1.73$$

$$1.73 = \frac{x}{13}$$

$$x = (1.73) \times (13) = 22.49$$


Using Pythagoras Theorem to find the length of the longest side (hypotenuse)



$$c^2 = a^2 + b^2$$

$$c^2 = 3^2 + 4^2$$

$$c^2 = 9 + 16$$

$$c^2 = 25$$

$$c = \sqrt{25}$$

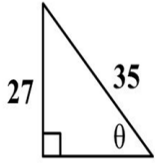
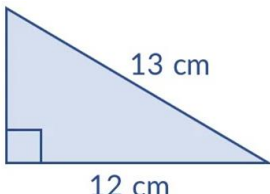
$$c = 5$$

When finding angle measures use the inverse of the trigonometric functions:  $\tan^{-1}$ ,  $\sin^{-1}$ ,  $\cos^{-1}$

Using Pythagoras Theorem to find the length of a shorter side

Example:  $\sin \theta = \frac{27}{35}$

$$\theta = \sin^{-1}\left(\frac{27}{35}\right)$$

$$\theta \approx 50^\circ$$



$$a = \sqrt{13^2 - 12^2}$$


$$a = \sqrt{169 - 144}$$

$$a = \sqrt{25}$$


$$a = 5 \text{ cm}$$

We can use these to work out missing sides and angles in **right-angled triangles**.


SOH


 $\sin(\theta) = \frac{O}{H}$

CAH


 $\cos(\theta) = \frac{A}{H}$

TOA


 $\tan(\theta) = \frac{O}{A}$

## CORE

## GOOD TO KNOW...

## HOW TO....

**Similarity** - Two shapes are Similar when one can become the other after an enlargement, reflection, translation or rotation. Corresponding angles are equal and corresponding sides are all in the same ratio.

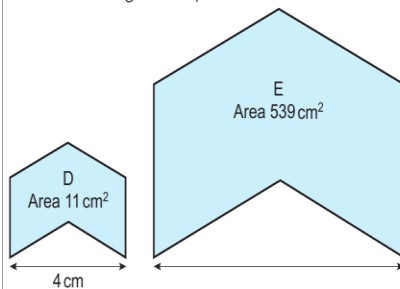
**Enlargement** - A type of transformation where we change the size of the original shape to make it bigger or smaller.

**Scale Factor** - The ratio between the scale of a given original object and a new object, which is its representation but of a different size (bigger or smaller).

**Congruent** - Two shapes are congruent if they have the same shape and size, or when one shape can be rotated or reflected to fit exactly on the other.

- When a linear scale factor is  $k$ 
  - Lengths are multiplied by  $k$
  - Area is multiplied by  $k^2$
  - Volume is multiplied by  $k^3$

Shape D is similar to shape E.  
Calculate the length of shape E.



$$\text{Area scale factor} = \frac{539}{11} = 49 = k^2$$

$$k = \sqrt{49} = 7$$

Shape E has length  $7 \times 4 = 28$  cm

In an enlargement by scale factor  $k$ , the area is enlarged by scale factor  $k^2$ .

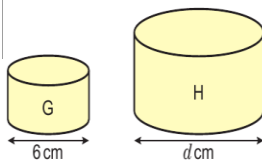
$k$  is the linear scale factor.

Cylinders G and H are similar.

The diameter of G is 6 cm.

The volume of G is  $108 \text{ cm}^3$ . The volume of H is  $256 \text{ cm}^3$ .

Work out the diameter  $d$  of cylinder H.



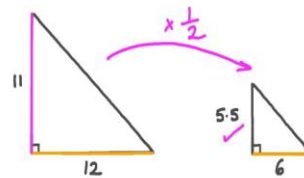
$$\text{Volume scale factor} = \frac{\text{large}}{\text{small}} = \frac{256}{108} = \frac{64}{27} = k^3$$

$$k = \sqrt[3]{\frac{64}{27}} = \frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \frac{4}{3}$$

$$d = \frac{4}{3} \times 6 = 8 \text{ cm}$$

In an enlargement by scale factor  $k$ , the volume is enlarged by scale factor  $k^3$ .

In the figure, given that the two triangles are similar, what is the scale factor that would take you from the larger triangle to the smaller triangle?



$$\text{Scale factor} = \frac{\text{new length}}{\text{original length}}$$

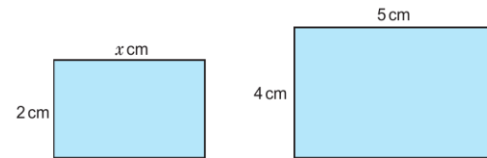
$$\text{scale factor} = \frac{6}{12} = \frac{1}{2}$$

$$11 \times \frac{1}{2} = \frac{11}{2} = 5.5$$

$$\boxed{\frac{1}{2}} \checkmark$$

- Corresponding angles are congruent
- Corresponding sides are in proportion

These two rectangles are similar. Find the missing length  $x$  in the smaller rectangle.



$$\text{ratio of lengths: } \frac{x}{5}$$

$$\text{ratio of widths: } \frac{2}{4} = \frac{1}{2}$$

$$\frac{\text{small}}{\text{large}} = \frac{1}{2} = \frac{x}{5}$$

$$2x = 5$$

$$x = \frac{5}{2} = 2.5 \text{ cm}$$

Write the ratio  $\frac{\text{small}}{\text{large}}$  for the lengths and the widths.

Write an equation to solve for  $x$ .

Measure	Multiplier or Divider
Length	Scale factor
Area	(Scale factor) <sup>2</sup>
Volume	(Scale factor) <sup>3</sup>

## CORE

## GOOD TO KNOW...

## HOW TO....

**Probability** - how likely something is to happen

- Probability is either given as a fraction, decimal or a percentage
- **Probability adds up to 1**
- The probability of an event not happening, is one minus the probability of the event happening.

Probability =  $\frac{\text{number of successful outcomes}}{\text{Total number of possible outcomes}}$

**Sample Space Diagram** shows all the possible outcomes of two events.

**Tree Diagrams** - shows two or more events and their probabilities.

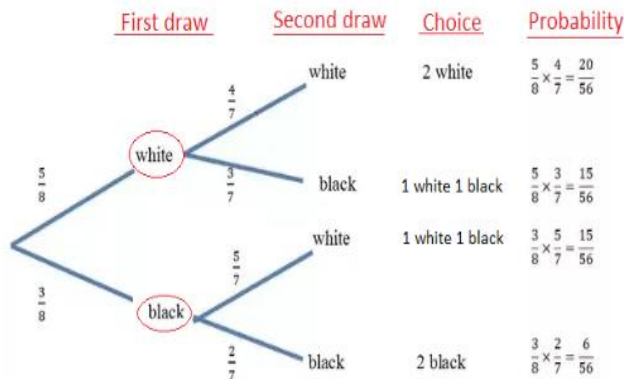
**Venn Diagrams** - Probabilities can be calculated using venn diagrams.

In a probability experiment a trial is repeated many times and the outcomes recorded. The relative frequency of an outcome is called the **experimental probability**.

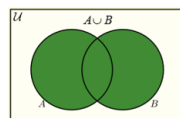
If one event depends on the other, the two events are **dependent**.

Two events are **independent** if one event does not affect the probability of the other. To find the probability of independent event, multiply the two probabilities together.

Two events are **mutually exclusive** if they cannot happen at the same time. When events are mutually exclusive, you can add their probabilities together.

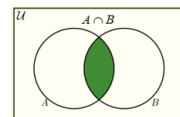


### Venn Diagrams



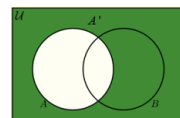
**A union B**

Elements that belong to either A or B or both.



**A intersect B**

Elements that belong to both A and B.



**A complement**

Elements that don't belong to A.

## Probability

### NOT

Independent event not occurring

1 minus the probability of occurrence

$$P = 1 - P(A)$$

What is the probability of **not** rolling a 1 on a die?

$$P = 1 - P_1 = 1 - \frac{1}{6} = \frac{5}{6}$$

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	<b>10</b>
5	6	7	8	9	<b>10</b>	11
6	7	8	9	<b>10</b>	11	12

Probability of getting a total of ten =  $\frac{3}{36}$

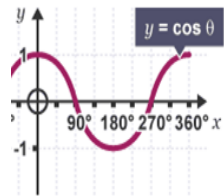
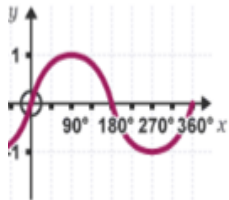
## CORE

## GOOD TO KNOW...

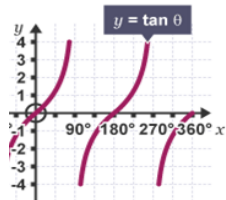
## HOW TO....

- Lower bound: a value that is less than or equal to every element of a set of data.
- Upper bound: a value that is greater than or equal to every element of a set of data.
- The graph of sine

The graph of cosine



- The graph of tangent



- Graphs can be transformed which means they look the same but are shifted or reflected in some way

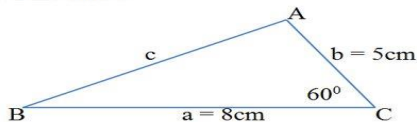
$$\text{Area of triangle} = \frac{1}{2} ab \sin C$$

$$\text{Area} = \frac{1}{2} (3)(4) \sin 55^\circ$$

$$= 4.9149\dots$$

$$= 4.9 \text{ cm}^2$$

1. Find side  $c$



Substitute  $a = 8$ ,  $b = 5$  and  $C = 60$  in the equation:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

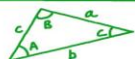
$$c^2 = 8^2 + 5^2 - 2 \times 8 \times 5 \times \cos 60 \quad (\text{do this on your calculator!})$$

$$c^2 = 49$$

$$c = 7$$

Find the size of angle  $A$ , in degrees, of the triangle shown.

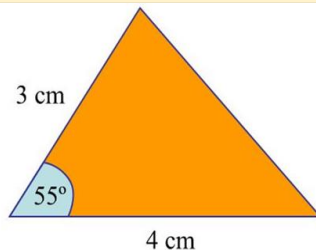
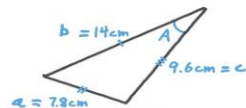
*The Cosine Rule*



$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

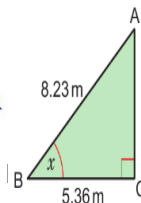
$$\cos^{-1} \left( \frac{a^2 - b^2 - c^2}{-2bc} \right) = A$$

$$A = \cos^{-1} \left( \frac{(7.8 \text{ cm})^2 - (14 \text{ cm})^2 - (9.6 \text{ cm})^2}{-2(14 \text{ cm})(9.6 \text{ cm})} \right) = \boxed{32^\circ}$$



In this diagram, the measurements are correct to 3 significant figures.

- Find the upper and lower bounds for the value of  $x$ , to 3 decimal places.
- Give the value of  $x$  to a suitable level of accuracy.



a AB: upper bound = 8.235 m, lower bound = 8.225 m

BC: upper bound = 5.365 m, lower bound = 5.355 m

$$\text{The upper bound for } \cos x = \frac{5.365}{8.225} = 0.6522796353$$

So  $x = 49.286^\circ$  (3 d.p.)

$$\text{The lower bound for } \cos x = \frac{5.355}{8.235} = 0.6502732240$$

So  $x = 49.438^\circ$  (3 d.p.)

So the upper bound for  $x$  is  $49.438^\circ$  and the lower bound is  $49.286^\circ$

b  $49.438^\circ = 49.4$  (1 d.p.)

$49.286^\circ = 49.3$  (1 d.p.)

$= 49^\circ$  (nearest degree)

$= 49^\circ$  (nearest degree)

$x = 49^\circ$  (to the nearest degree)

Find the upper and lower bounds of the lengths of AB and BC.

The upper bound of a fraction =  $\frac{\text{upper bound of the numerator}}{\text{lower bound of the denominator}}$   
Write down all the figures in your calculator display.

Use  $\cos^{-1}$  on your calculator.

The lower bound of a fraction =  $\frac{\text{lower bound of the numerator}}{\text{upper bound of the denominator}}$

You could write the answer as  $49.286^\circ \leq x < 49.438^\circ$

Round the upper and lower bounds to 1 d.p. Do they both give the same value?

Round to the nearest degree they both give the same value.

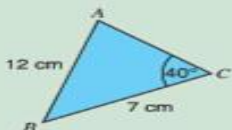
Find the size of angle  $A$  in this triangle.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{7} = \frac{\sin 40}{12}$$

$$\sin A = \frac{7 \times \sin 40}{12} = 0.375$$

$$\therefore A = \sin^{-1} 0.375 = 22^\circ$$



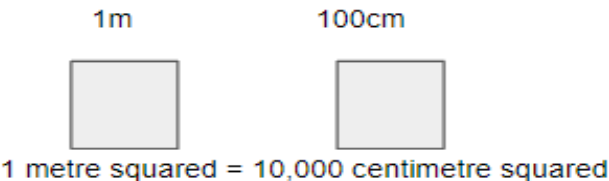


CORE	GOOD TO KNOW...	HOW TO....
------	-----------------	------------

- Original amount is always 100%
- Percent is per 100

- 1cm = 10mm
- 1m = 100cm
- 1km = 1000m
- 1kg = 1000g
- 1l = 1000ml

- Inverse - opposite, e.g. inverse of add is subtract
- Proportion - a mathematical comparison between two numbers - if the ratios that the two numbers increase/decrease are the same this is direct proportion
- Ratio - The relationship in quantity, amount or size



**Increase £50 by 60%.**

$$160\% \times £50 = 1.6 \times £50 = £80$$

**Increase £86 by 7%.**

$$107\% \times £86 = 1.07 \times £86 = £92.02$$

Calculate the interest on borrowing £40 for 3 years if the compound interest rate is 5% per year.

- Year 1:  $£40 + 5\% = £40 + £2 = £42$
- Year 2:  $£42 + 5\% = £42 + £2.10 = £44.10$
- Year 3:  $£44.10 + 5\% = £44.10 + £2.21 = £46.31$

$$PV \times (1 + r)^n = FV$$

Present Value      Interest Rate (as a decimal)      Number of Periods      Future Value

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

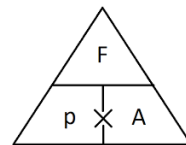
$$\text{Mass} = \text{Density} \times \text{Volume}$$

**Decrease £65 by 20%.**

$$80\% \times £65 = 0.8 \times £65 = £52$$

**Decrease £320 by 3%.**

$$97\% \times £320 = 0.97 \times £320 = £310.40$$



## GOOD TO KNOW...

## HOW TO....

Example: A pair of socks went from \$5 to \$6, what is the percentage change?

Answer (Method 1):

- Step 1: \$5 to \$6 is a \$1 increase
- Step 2: Divide by the old value:  $\$1/\$5 = 0.2$
- Step 3: Convert 0.2 to percentage:  $0.2 \times 100 = 20\%$  rise.

If a driver has travelled 180 miles and it took them 3 hours to make that distance, then to work out their speed you would take:

$$180 \text{ miles} / 3 \text{ hours} \rightarrow 180 / 3 = 60$$

So the driver's speed would be 60mph.

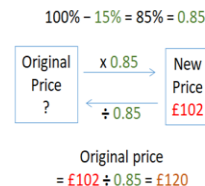
## Percent Change

$$\text{Percent Change} = \frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100\%$$

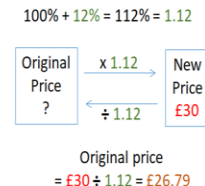
If the result is positive, it is an increase.  
If the result is negative, it is a decrease.

## Reverse Percentage

A jacket costs £102 after a discount of 15%.  
What is the original price of the jacket?



The price of a ticket costs £30 inclusive of 12% tax.  
What is the pre-tax cost of the ticket?





## CORE

- Compare data sets using averages
- Mode is the piece of data which appears the most
- Range is the difference between the largest and smallest piece of data
- Median is the middle value of an ordered set of data
- Mean is the average found by adding all the values and dividing by the total number of values
- A data set is a collection of data

### Keywords

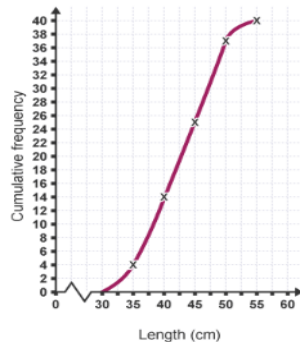
- Sampling - using a portion of a total population to represent the full population
- Median - list numbers in numerical order and find the middle value
- Range - the biggest value minus the smallest value
- Outlier - a data point which doesn't fit the trend of the rest of the data

## GOOD TO KNOW...

Length (cm)	Frequency	Cumulative frequency
$30 < l \leq 35$	4	4
$35 < l \leq 40$	10	14 ( $4 + 10 = 14$ )
$40 < l \leq 45$	11	25 ( $14 + 11 = 25$ )
$45 < l \leq 50$	12	37 ( $25 + 12 = 37$ )
$50 < l \leq 55$	3	40 ( $37 + 3 = 40$ )

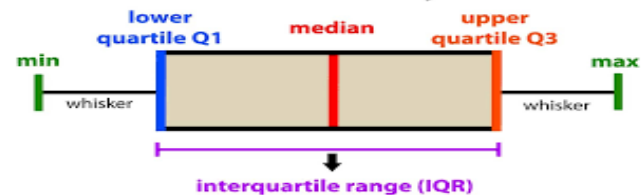
A cumulative frequency diagram is drawn by plotting the cumulative frequency against the **upper class boundary** of the respective group. The upper class boundaries for this table are 35, 40, 45, 50 and 55.

Cumulative frequency is plotted on the vertical axis and length is plotted on the horizontal axis.



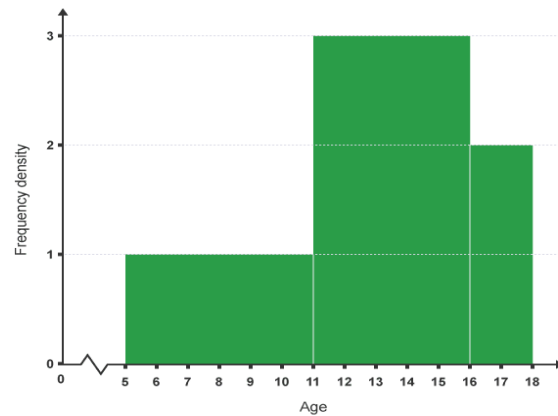
## HOW TO....

### introduction to data analysis: Box Plot



Age	Frequency	Class width	Frequency density
5-10	6	6 (5, 6, 7, 8, 9 and 10 are in this category)	$6 \div 6 = 1$
11-15	15	5	$15 \div 5 = 3$
16-17	4	2	$4 \div 2 = 2$

Once the frequency densities of the numbers are known, the histogram can be drawn.



# Year 10 Term 5 Maths Knowledge Organiser [F Unit 15 - Constructions, loci and bearings]

## CORE

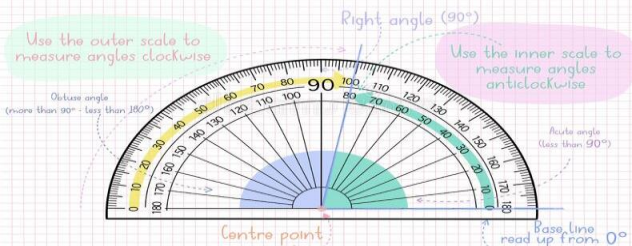
## GOOD TO KNOW...

## HOW TO....



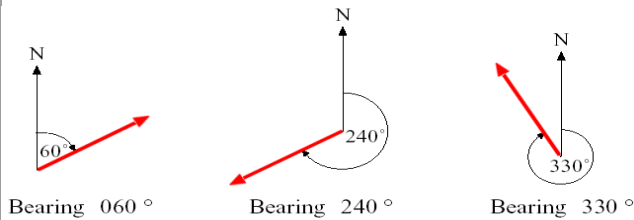
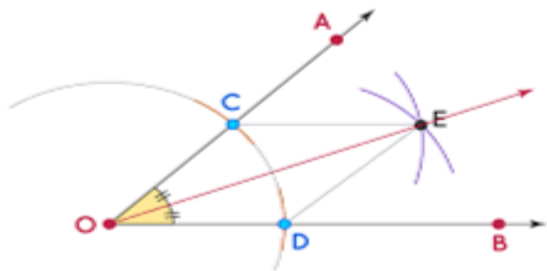
- Bearings are always measured from North, in a clockwise direction and are written using 3 figures, e.g. 045 degrees
- Measure using a ruler and protractor accurately

## HOW TO USE A PROTRACTOR

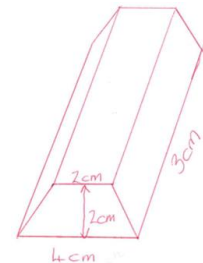
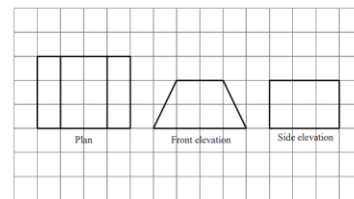


- Identify cube, cuboid and cylinder
- 2D - a 2 dimension shape (flat, e.g. square, circle)
- 3D - a 3 dimensional shape (solid, e.g. cube, cylinder)
- Bisector - a line which divides a line or angle in half exactly
- Loci - a set of points with the same property e.g. within 3cm of a point - you would draw a circle using compasses set at 3cm from the point given
- Perpendicular - at 90 degrees to a given line
- Congruent - a shape that is exactly the same shape and size
- Similar - a shape with the same size angles with all corresponding sides in proportion

- Identify and use SSS, ASA and SAS
- Plan is the view of a 3D shape when looked at from above
- Elevation is the view of a 3D shape when looked at from the front or the side
- Can show on a diagram a region by using given parameters using loci
- Can bisect an angle or line using compasses



The diagram shows the plan, front elevation and side elevation of a solid shape, drawn on a centimetre grid.



In the space below, draw a sketch of the solid shape. Give the dimensions of the solid on your sketch.

Scale = 1 cm : 5 km

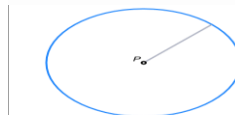
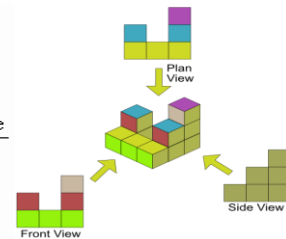
∴ Scale factor = 5

Actual distance = 14 km

$$\text{Map distance} = \frac{\text{Actual distance}}{\text{Scale factor}}$$

$$= \frac{14}{5}$$

$$= 2.8$$



The set of points a given distance from a single point, P, is a circle

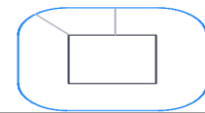


A locus, set at a fixed distance from a line, gives a racetrack pattern: a semi-circle around then end points, with parallel lines connecting them

A locus that is equidistant from two points X and Y is described with a straight line. It can be constructed using a pair of arcs.



A locus equidistant from two intersecting lines is an angle bisector



A locus traced around a square gives a lozenge shape, with a quarter circle at each corner.

## CORE

## GOOD TO KNOW...

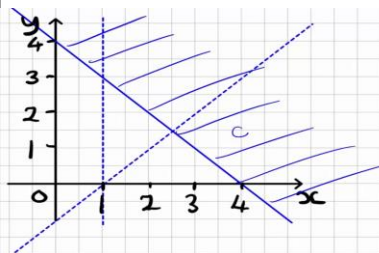
## HOW TO....

- Simultaneous equations are two equations with shared variables
- Simultaneous equations can be solved algebraically or graphically
- Expand brackets
- Inequalities can be solved algebraically or graphically
  - $<$  means less than
  - $>$  means greater than
  - $\leq$  means less than or equal to
  - $\geq$  means greater than or equal to
- Identify the turning point on a quadratic graph

### Keywords

- Expand - multiply term outside the bracket by all terms inside the brackets to eliminate brackets
- Solve - find the exact value of the unknown variable
- Equation - an expression that contains an equals sign
- Quadratic - one unknown term is squared
- Root - a solution to a quadratic or cubic equation. There can be more than one root
- Coefficient - a number that is being multiplied by the variable e.g.  $2x + 6$  2 is the coefficient which  $x$  is multiplied by
- Turning point - the point where a graph changes direction

- Expand more than one set of brackets
- Factorise and solve a quadratic equation
- Substitute values into an equation to create a table of values to plot a quadratic or cubic graph
- Solve a quadratic equation graphically (find the roots)
- Recognise linear, quadratic, cubic and reciprocal graphs
- Be able to complete the square to find turning points algebraically
- Solve simultaneous equations graphically
- Draw inequalities on a graph and the use to show a region which satisfies the inequalities
- Know that when representing inequalities on a graph
  - $< >$  means a dotted line
  - $\leq \geq$  means a solid line



$$\begin{aligned}x + y &= 4 \\y &= x - 1 \\x &= 1\end{aligned}$$

$$\begin{aligned}x + y &\leq 4 \\y &> x - 1 \\x &> 1\end{aligned}$$

Find the solution to the equation  $x^3 + 5x = 20$  using the initial value  $x_0 = 2$ , giving the answer to 3 decimal places.

First, rearrange the equation to leave  $x$  on its own on one side of the equation.

One way to do this is:

$$x^3 + 5x = 20$$

$$x^3 = 20 - 5x$$

$$x = \sqrt[3]{20 - 5x}$$

To solve the equation, use the iterative formula

$$x_{n+1} = \sqrt[3]{20 - 5x_n}$$

We are given the initial value  $x_0 = 2$

Substituting this into the iterative formula gives

$$x_1 = \sqrt[3]{20 - 5 \times 2} = \sqrt[3]{10} = 2.154 \dots$$

Substituting iteratively gives:

$$x_2 = \sqrt[3]{20 - 5 \times 2.154} = \sqrt[3]{9.227 \dots} = 2.097 \text{ (3 dp)}$$

$$x_3 = \sqrt[3]{20 - 5 \times 2.097} = \sqrt[3]{9.512 \dots} = 2.118 \text{ (3 dp)}$$

$$x_4 = \sqrt[3]{20 - 5 \times 2.118} = \sqrt[3]{9.405 \dots} = 2.111 \text{ (3 dp)}$$

$$x_5 = \sqrt[3]{20 - 5 \times 2.111} = \sqrt[3]{9.445 \dots} = 2.114 \text{ (3 dp)}$$

$$x_6 = \sqrt[3]{20 - 5 \times 2.114} = \sqrt[3]{9.430 \dots} = 2.113 \text{ (3 dp)}$$

$$x_7 = \sqrt[3]{20 - 5 \times 2.113} = \sqrt[3]{9.436 \dots} = 2.113 \text{ (3 dp)}$$

Since  $x_6$  and  $x_7$  give the same value to 3 decimal places, the iteration stops. The solution to the equation  $x^3 + 5x = 20$  is 2.113 to 3 decimal places.

# Year 10 Term 5 Maths Knowledge Organiser [F Unit 16 - Quadratic equations & graphs TKAT

## CORE

- Expand a single bracket
- Factorise expressions
- Solve linear equations
- Simplify an expression by collecting like terms

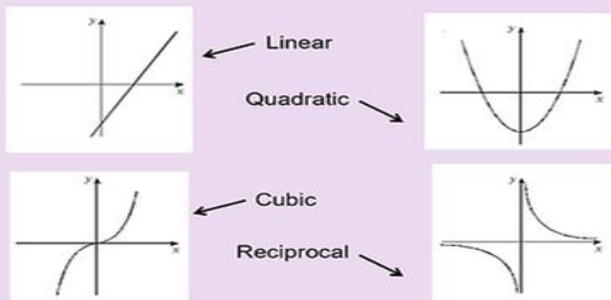
### Keywords

- Expand - multiply term outside the bracket by all terms inside the brackets to eliminate brackets
- Solve - find the exact value of the unknown variable
- Equation - an expression that contains an equals sign
- Quadratic - one unknown term is squared
- Root - a solution to a quadratic or cubic equation. There can be more than one root
- Substitute - replace the given letter with the given value
- Factor - a number that divides a number without a remainder ie. 5 is a factor of 10.
- Factorise - find hcf and put brackets back into expression - reverse of expanding
- Coefficient - a number that is being multiplied by the variable e.g.  $2x + 6$  2 is the coefficient which x is multiplied by
- Simplify - collect like terms
- Reciprocal - the inverse if a number e.g. the reciprocal of 2 is  $\frac{1}{2}$
- Turning point - the point where a graph changes direction

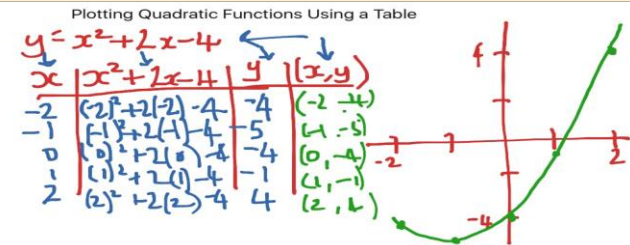
## GOOD TO KNOW...

- Expand double brackets and recognise that double brackets form a quadratic expression
- Factorise a quadratic expression and solve
- Use algebra to represent side lengths and then use to calculate perimeter/area
- Recognise linear, quadratic, cubic and reciprocal graphs
- Substitute values into an equation to create a table of values to plot a quadratic graph
- Know what roots are and be able to find the roots of an equation from a graph
- Solve quadratic equation graphically

### Types of Graphs



## HOW TO....



## Factorise $x^2 + 11x + 24$

$$x^2 + 11x + 24$$

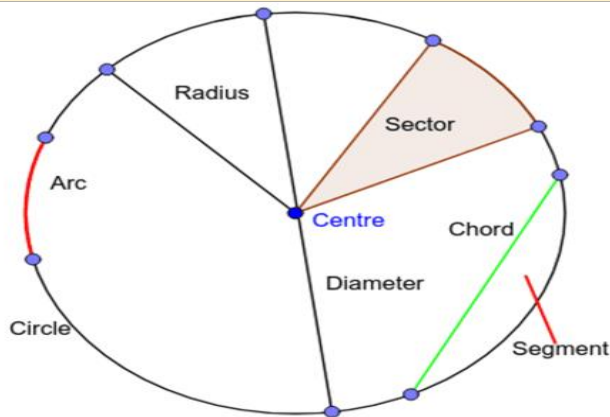
Diagram showing the factorisation process. The expression  $x^2 + 11x + 24$  is shown with  $+24$  and  $+11$  circled. A box contains  $+24$  and  $+11$  with arrows pointing to the  $x$  and  $+$  terms respectively. Below this, a box shows  $+3$  and  $+8$  with arrows pointing to the  $+11$  and  $+24$  terms respectively.

Find two numbers that multiply to get +24 and add to get +11

Final answer:  $(x + 8)(x + 3)$

Diagram showing the expansion of  $(x + 1)(x + 2)$  to  $x^2 + 3x + 2$ . The expression  $(x + 1)(x + 2)$  is shown with arrows indicating the FOIL method: F (x to x), O (x to 2), I (1 to x), L (1 to 2). The result is  $x^2 + 2x + 1x + 2 = x^2 + 3x + 2$ .

## CORE



- Radius is half the diameter

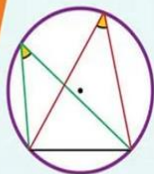
- Isosceles triangles have 2 equal sides and 2 equal angles
- Right angled triangles have one right angle

### Keywords

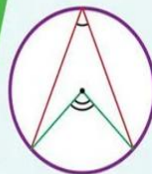
- Tangent - a line which is at 90 degrees to the radius it joins with at the circumference
- Segment - a region of a circle created by a chord
- Chord - a line joining two points on the circumference which doesn't pass through the centre
- Arc - a curve joining two points on the circumference of a circle
- Sector - a region of a circle bounded by two radii and an arc

## GOOD TO KNOW...

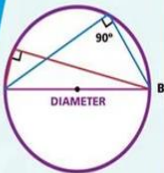
Angles in the same segment and standing on the same chord are always equal.



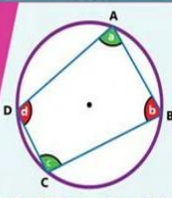
The angle at the centre of a circle is twice the angle at the circumference (outer edge).



The angle in a semi-circle is always 90°.

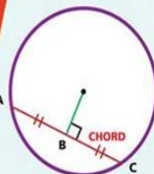


ABCD is a cyclic quadrilateral - all vertices lie on the circumference of the circle. Diagonally opposite angles add up to 180°.

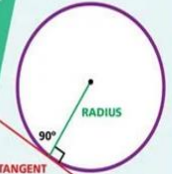


$$b + d = 180^\circ \quad a + c = 180^\circ$$

A line drawn from the centre of a circle to the mid-point of a chord is perpendicular (at 90°) to the chord.

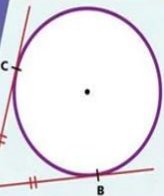


The angle between the tangent and the radius is always 90°.

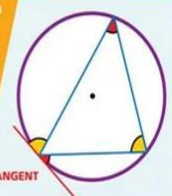


Tangents from a common point (A) to a circle are always equal in length.

$$AB = AC$$

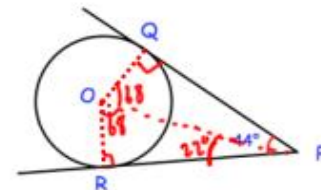


The angle between the tangent and the side of the triangle is equal to the interior opposite angle.



## HOW TO....

Q and R are points on a circle, centre O. PQ and PR are tangents to the circle.

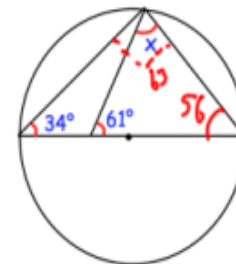


Work out the size of angle POR.

$$\begin{array}{r} 90 \\ + 22 \\ \hline 112 \end{array} \quad \begin{array}{r} 180 \\ - 112 \\ \hline 68 \end{array}$$

68

AB is the diameter of a circle.



Work out the value of x.

$$\begin{array}{r} 34 \\ + 90 \\ \hline 124 \end{array} \quad \begin{array}{r} 180 \\ - 124 \\ \hline 56 \end{array}$$

63



CORE	GOOD TO KNOW...	HOW TO....
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## Circle

**Circumference** -  $\pi d$

**Area** -  $\pi r^2$

- Measure in squared units, e.g.  $\text{cm}^2$ .

**Volume**

- Cylinder = area of circle (front face) x length
- Measure in cubic units, e.g.  $\text{cm}^3$ .

$\pi = 3.14\dots$  (2d.p.)

- **Area** : Square/Rectangle = length x width
- **Area** : Triangle =  $\frac{1}{2} \times \text{base} \times \text{height}$

## Keywords

**Circumference** - The distance around the circle.

**Area** - The space inside a 2D shape.

**Diameter (d)** - A straight line that runs from one side of a circle to another and passes through the center.

**Radius (r)** - A straight line from the centre of the circle to the edge. Radius is half the diameter.

**Volume** - The volume of a 3D shape is the amount of space inside it.

**Prism** - A 3D shape which has the same cross section throughout, e.g. cylinder.

**Surface Area** - The amount of space covering the outside of a 3D shape.

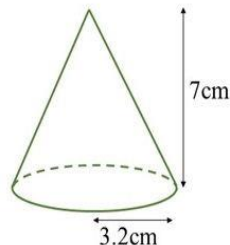
**Arc** - a curve joining two points on the circumference of a circle

**Sector** - a region of a circle bounded by two radii and an arc.

## GOOD TO KNOW...

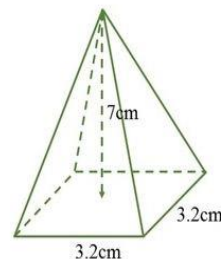
### Example

Which has the greatest volume?



$$\text{Base Area} = \pi \times 3.2^2 = 32.17$$

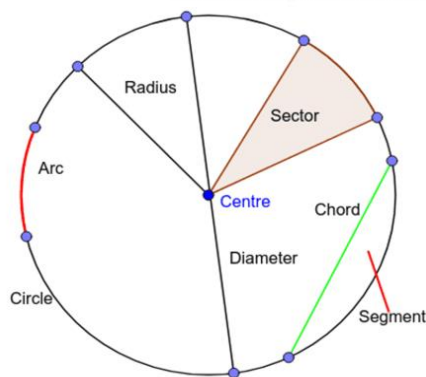
$$\begin{aligned} \text{Volume} &= \frac{1}{3} \times 32.17 \times 7 \\ &= \underline{75.06 \text{ cm}^3} \end{aligned}$$



$$\text{Base Area} = 3.2 \times 3.2 = 10.24$$

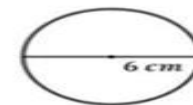
$$\begin{aligned} \text{Volume} &= \frac{1}{3} \times 10.24 \times 7 \\ &= \underline{23.89 \text{ cm}^3} \end{aligned}$$

Cone has the greatest volume



## HOW TO....

### EXAMPLE (CIRCUMFERENCE)

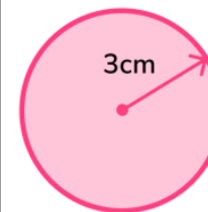


$$\begin{aligned} C &= \pi d \\ &= 3.142 \times 6 \text{ cm} \\ &= 18.85 \text{ cm} \end{aligned}$$



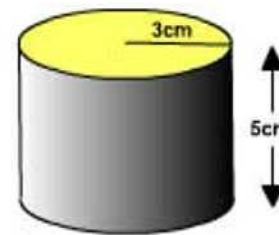
$$\begin{aligned} C &= 2\pi r \\ &= 2 \times 3.142 \times 4 \text{ cm} \\ &= 25.14 \text{ cm} \end{aligned}$$

What is the area of a circle with radius 3cm?



$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= \pi \times 3^2 \\ &= 9\pi \text{ cm}^2 \\ &= 28.3 \text{ cm}^2 \text{ (1.d.p.)} \end{aligned}$$

## Volume of cylinders



$$\begin{aligned} \text{Volume} &= \pi r^2 h \\ &= \pi \times 3^2 \times 5 \\ &= \pi \times 9 \times 5 \\ &= 141.37 \text{ cm}^3 \end{aligned}$$



CORE	GOOD TO KNOW...	HOW TO....
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- Changing the subject of an equation means rearranging that equation so that letter appears on its own (i.e.  $y = 3x$  shows that  $y$  is the subject)
- Algebraic fractions can be worked out the same way as normal fractions
  - Add/ subtract fractions by finding equivalent fractions with the same denominator
  - Multiply fractions by multiplying the numerators together and the denominators together
  - Divide fractions by following the KFC rule: keep the first fraction as it is, flip the second fraction around so the numerator becomes the denominator and change the sign from a divide to times.
- Surds are square roots that cannot be worked out to produce an exact value (i.e.  $\sqrt{5}$  is in surd form as it has a square root sign)

Make  $x$  the subject of the formula  $P = d\sqrt{\frac{x}{y}}$

$\frac{P}{d} = \sqrt{\frac{x}{y}}$  ————— Divide both sides by  $d$ .

$\frac{P^2}{d^2} = \frac{x}{y}$  ————— Square both sides.

$\frac{yP^2}{d^2} = x$  or  $x = \frac{yP^2}{d^2}$

Simplify fully  $\frac{x^2 + 5x + 4}{x^2 - 3x - 28}$

$\frac{x^2 + 5x + 4}{x^2 - 3x - 28} = \frac{(x+1)(x+4)}{(x-7)(x+4)}$  ————— Factorise the numerator and denominator.

$= \frac{x+1}{x-7}$  ————— Divide the numerator and denominator by the common factor  $(x+4)$ .

Find the inverse function of  $x \rightarrow 5x - 1$

$x \rightarrow \boxed{\times 5} \rightarrow \boxed{-1} \rightarrow 5x - 1$

Write the function as a function machine.

$\frac{x+1}{5} \leftarrow \boxed{\div 5} \leftarrow \boxed{+1} \leftarrow x$

Reverse the function machine to find the inverse function. Start with  $x$  as the input.

The inverse function of  $x \rightarrow 5x - 1$  is  $x \rightarrow \frac{x+1}{5}$

A **proof** is a logical argument for a mathematical statement. To prove a statement is true, you must show that it will be true in *all* cases.

To prove a statement is not true you can find a **counter-example** - an example that does not fit the statement.

Show that  $(x+4)^2 - 7 \equiv x^2 + 8x + 9$

LHS  $= (x+4)^2 - 7 \equiv (x+4)(x+4) - 7 = x^2 + 8x + 16 - 7 = x^2 + 8x + 9$

Expand the brackets on the left-hand side (LHS).

RHS  $= x^2 + 8x + 9$

So LHS = RHS and  $(x+4)^2 - 7 \equiv x^2 + 8x + 9$  ————— Aim to show that LHS = RHS.

Simplify  $\frac{x}{5} + \frac{x}{3}$

LCM of 5 and 3 is 15 ————— Find the LCM of the denominators.

$\frac{x}{5} = \frac{3x}{15}$       $\frac{x}{3} = \frac{5x}{15}$  ————— Write both fractions with the same denominator.

$\frac{3x}{15} + \frac{5x}{15} = \frac{8x}{15}$  ————— Add the fractions.

To rationalise the fraction  $\frac{1}{a\sqrt{b}}$ , multiply by  $\frac{\sqrt{b}}{\sqrt{b}}$

To rationalise the fraction  $\frac{1}{a \mp \sqrt{b}}$ , multiply by  $\frac{a \pm \sqrt{b}}{a \pm \sqrt{b}}$

Write  $\frac{7}{x+2} - \frac{3}{x+3}$  as a single fraction in its simplest form.

Common denominator  $= (x+2)(x+3)$  ————— Find a common denominator.

$\frac{7(x+3)}{(x+2)(x+3)} - \frac{3(x+2)}{(x+2)(x+3)}$

Convert each fraction to an equivalent fraction with the common denominator  $(x+2)(x+3)$ .

$= \frac{7(x+3) - 3(x+2)}{(x+2)(x+3)}$  ————— Subtract the fractions.

$= \frac{7x + 21 - 3x - 6}{(x+2)(x+3)} = \frac{4x + 15}{(x+2)(x+3)}$  ————— Expand the brackets in the numerator, then simplify.



# Knowledge Organisers

## Year 11

# Year 11 Term 1 Maths Knowledge Organiser [F unit 18 - Fractions, Indices & Standard Form]



## CORE

## GOOD TO KNOW...

## HOW TO....

**Mixed Number** - A number consisting of a whole number and a proper fraction.

**Improper Fraction** - A fraction whose numerator is larger than the denominator.

### Mixed Numbers Calculations

When multiplying or dividing mixed numbers change to an improper (top heavy) fraction first

### Indices

- $2 \times 2 \times 2 \times 2$  can be written  $2^4$
- When multiplying powers add the powers  
e.g.  $6^4 \times 6^7 = 6^{11}$  OR  $a^3 \times a^5 = a^8$
- When dividing powers subtract the powers  
e.g.  $6^8 \div 6^5 = 6^3$  OR  $a^9 \div a^5 = a^4$
- When in brackets multiply the powers  
e.g.  $(8^4)^3 = 8^{12}$  OR  $(x^5)^2 = x^{10}$
- Any number to the power of zero is 1
- The reciprocal of any number is 1 divided by the number eg: the reciprocal of 3 is  $\frac{1}{3}$
- The reciprocal of a number is found by raising the number to the power of -1
- To find a negative power, find the reciprocal and raise to the positive power

**Standard Form** - is used to write very large or very small numbers

$$a \times 10^n$$

Where  $1 \leq a < 10$  and  $n$  is an integer.

## Laws of indices

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{m \times n}$$

$$10^3 = 1000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$10^{-1} = 0.1$$

$$10^{-2} = 0.01$$

$$10^{-3} = 0.001$$

### Examples

$$a \times 10^n$$

Work out the value of  $(6.4 \times 10^7) \times (2 \times 10^{-3})$   
Give your answer in standard index form.

$$= 6.4 \times 2 \times 10^7 \times 10^{-3}$$

Law of indices

Write the following in standard index form:

32 000 000

0.000 005 74

$$= 3.2 \times 10\,000\,000$$

$$= 5.74 \times 0.000\,001$$

$$= 3.2 \times 10^7$$

$$= 5.74 \times 10^{-6}$$

Write the following as ordinary numbers:

$$8.35 \times 10^{-3}$$

$$2.9 \times 10^6$$

$$= 8.35 \times 0.001$$

$$= 2.9 \times 1\,000\,000$$

$$= 0.008\,35$$

$$= 2\,900\,000$$

$$\begin{array}{r} + \\ 2 \\ \times \\ \hline \end{array} \begin{array}{r} 3 \\ 4 \\ \hline \end{array} = \frac{(4 \times 2) + 3}{4} = \frac{8 + 3}{4} = \frac{11}{4}$$

Mixed Number

Improper Fraction

# Year 11 Term 1 Maths Knowledge Organiser [H unit 18 - Vectors and Geometric Proof]



## CORE

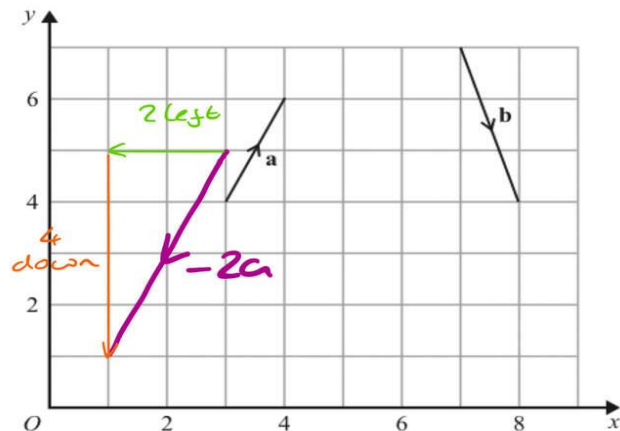
Vector arithmetic:  
Where  $\mathbf{a}$  is the vector  $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$$2\mathbf{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \quad 3\mathbf{a} = \begin{pmatrix} 9 \\ 12 \end{pmatrix} \quad 5\mathbf{a} = \begin{pmatrix} 15 \\ 20 \end{pmatrix}$$

We can add vectors by adding the two x components and adding the two y components together.

$$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

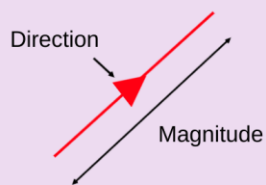
$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 + 2 \\ 4 + 7 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$



## GOOD TO KNOW...

Vectors can be represented as:

### Straight Lines



### Column Vectors

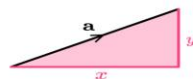
$$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

The **magnitude of a vector** is the length of a vector. It is also known as the **modulus** and is the absolute value of the vector.

The magnitude of the vector  $\mathbf{a}$  is written as  $|\mathbf{a}|$ .

To work out the magnitude of a vector we use Pythagoras' theorem.

E.g.  $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$



$$|\mathbf{a}| = \sqrt{x^2 + y^2}$$

$A$  is the point  $(3, 2)$  and  $B$  is the point  $(4, -1)$ .

(a) Write down as a column vector  $\overrightarrow{AB}$

$$\begin{pmatrix} 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

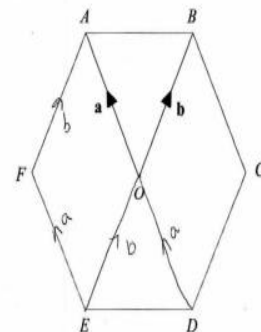
$C$  is the point  $(5, -2)$  and  $D$  is the point  $(2, 1)$ .

(b) Write down as a column vector  $\overrightarrow{CD}$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

## HOW TO....

$ABCDEF$  is a regular hexagon with centre  $O$ .



$$\overrightarrow{OA} = \mathbf{a}$$

$$\overrightarrow{OB} = \mathbf{b}$$

(a) Find, in terms of  $\mathbf{a}$ , the vector  $\overrightarrow{AD}$

(b) Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the vector  $\overrightarrow{AB}$

(c) Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ , the vector  $\overrightarrow{AF}$

$$-2\mathbf{a}$$

(1)

$$-\mathbf{a} + \mathbf{b}$$

(1)

$$-\mathbf{b}$$

(1)

# Year 11 Term 1 Maths Knowledge Organiser [F unit 19 - Congruence, similarity and vectors]



## CORE

Vector arithmetic:  
Where  $\mathbf{a}$  is the vector  $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$$2\mathbf{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \quad 3\mathbf{a} = \begin{pmatrix} 9 \\ 12 \end{pmatrix} \quad 5\mathbf{a} = \begin{pmatrix} 15 \\ 20 \end{pmatrix}$$

We can add vectors by adding the two x components and adding the two y components together.

$$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 + 2 \\ 4 + 7 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

### Keywords

**Similarity** - Two shapes are Similar when one can become the other after an enlargement, reflection, translation or rotation.

**Enlargement** - A type of transformation where we change the size of the original shape to make it bigger or smaller.

**Scale Factor** - The ratio between the scale of a given original object and a new object, which is its representation but of a different size (bigger or smaller).

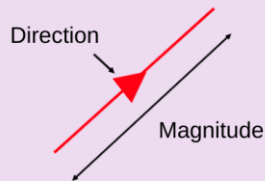
**Congruent** - Two shapes are congruent if they have the same shape and size, or when one shape can be rotated or reflected to fit exactly on the other.

**Vector** - A vector is a quantity that has both a magnitude and a direction

## GOOD TO KNOW...

Vectors can be represented as:

### Straight Lines



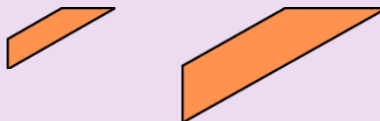
### Column Vectors

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

**Congruent shapes** have all sides and angles equal.

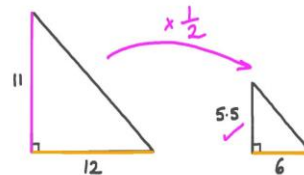


**Similar shapes** have all angles equal but one is an enlargement of the other.



## HOW TO....

In the figure, given that the two triangles are similar, what is the scale factor that would take you from the larger triangle to the smaller triangle?



$$\text{scale factor} = \frac{\text{new length}}{\text{original length}}$$

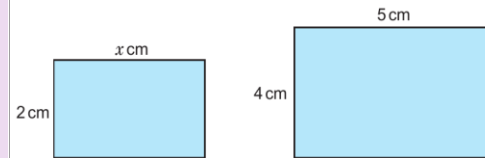
$$\text{scale factor} = \frac{6}{12} = \frac{1}{2}$$

- Corresponding angles are congruent
- Corresponding sides are in proportion

$$11 \times \frac{1}{2} = \frac{11}{2} = 5.5$$

$$\boxed{\frac{1}{2}} \quad \checkmark$$

These two rectangles are similar. Find the missing length  $x$  in the smaller rectangle.



$$\text{ratio of lengths: } \frac{x}{5}$$

$$\text{ratio of widths: } \frac{2}{4} = \frac{1}{2}$$

Write the ratio  $\frac{\text{small}}{\text{large}}$  for the lengths and the widths.

$$\frac{\text{small}}{\text{large}} = \frac{1}{2} = \frac{x}{5}$$

$$2x = 5$$

$$x = \frac{5}{2} = 2.5 \text{ cm}$$

Write an equation to solve for  $x$ .

## CORE

## GOOD TO KNOW...

## HOW TO....

**Direct proportion** means that one quantity increases at the same rate as the other.

- If one banana costs 20p, three bananas will cost 60p etc. The amount of bananas increase by x3 and the cost also increases by x3 so both are in direct proportion.

A quantity can be proportional to the square, cube or square root of another quantity.

**Inverse proportion** means that when one quantity increases, the other decreases.

- If it takes 2 decorators 6 hours to paint a room, it will take 4 decorators 3 hours to paint the same room. The number of decorators has doubled and the amount of time has halved.

$y$  is directly proportional to  $x$ .

When  $y = 20$ ,  $x = 8$

- Express  $y$  in terms of  $x$ .
- Find  $x$  when  $y = 35$ .

- $y \propto x$

So,  $y = kx$

$20 = k \times 8$

$k = 2.5$

$y = 2.5x$

$35 = 2.5 \times x$

$x = 14$
- Write  $y$  is directly proportional to  $x$ , using the symbol  $\propto$ .
- Write the equation using  $k$ .
- Substitute  $y = 20$  and  $x = 8$ . Solve to find  $k$ .
- Substitute the value of  $k$  back into the equation.
- Substitute  $y = 35$  into  $y = 2.5x$ .

$y$  is inversely proportional to  $x$ .

When  $y = 2$ ,  $x = 3$

- Write a formula for  $y$  in terms of  $x$ .
- Calculate the value of  $y$  when  $x = 8$

- $y \propto \frac{1}{x}$  so  $y = \frac{k}{x}$

$2 = \frac{k}{3}$  so  $k = 6$

$y = \frac{6}{x}$
  - $y = \frac{6}{8} = \frac{3}{4}$
- Write  $y$  is inversely proportional to  $x$  using the  $\propto$  symbol. Then write the equation using  $k$ .
- Substitute  $y = 2$  and  $x = 3$ . Solve to find  $k$ .
- Substitute  $k$  back into the equation.
- Substitute  $x = 8$  into your formula.

The sketch shows part of the graph  $y = ab^x$

The points with coordinates (0, 3) and (2, 12) lie on the graph.

Work out the values of  $a$  and  $b$ .

- $y = ab^x$

$3 = a \times b^0$

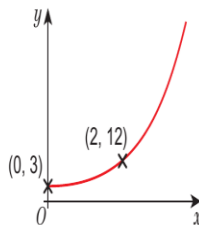
$3 = a \times 1$

$a = 3$
  - $y = 3b^x$

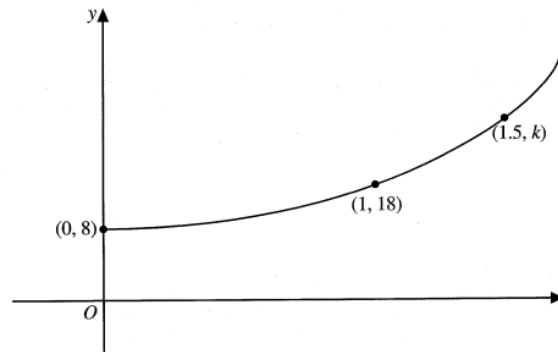
$12 = 3b^2$

$4 = b^2$

$b = 2$
- For the point (0, 3) substitute  $x = 0$  and  $y = 3$  into  $y = ab^x$
- $b^0 = 1$
- $a = 3$ , so the equation is  $y = 3b^x$
- For the point (2, 12), substitute  $x = 2$  and  $y = 12$  into  $y = 3b^x$



This sketch shows part of the graph with equation  $y = pq^x$  where  $p$  and  $q$  are constants.



The points with coordinates (0, 8), (1, 18) and (1.5,  $k$ ) lie on the graph.

Calculate the values of  $p$ ,  $q$  and  $k$ .

$$y = pq^x \quad (0, 8)$$

$$8 = p q^0$$

$$\underline{8 = p}$$

$$y = 8q^x \quad (1, 18)$$

$$18 = 8q$$

$$2 = \frac{18}{8}$$

$$= \frac{9}{4}$$

$$\underline{\underline{2 = \frac{9}{4}}}$$

$$y = 8\left(\frac{9}{4}\right)^x \quad (1.5, k)$$

$$k = 8\left(\frac{9}{4}\right)^{1.5}$$

$$= 8\left(\frac{9}{4}\right)^{\frac{3}{2}}$$

$$= 8\left(\frac{3}{2}\right)^3$$

$$= 8\left(\frac{27}{8}\right)$$

$$= \underline{\underline{27}}$$

$$\underline{\underline{p = 8}} \quad \underline{\underline{q = \frac{9}{4}}} \quad \underline{\underline{k = 27}}$$



## CORE

- A linear graph is a straight line
- Quadratic, cubic and reciprocal graphs are curved
- A quadratic equation contains a term in  $x^2$  but no higher power. It can also have  $x$  and number terms.
- A cubic contains a term in  $x^3$  but no higher power. It can also have terms in  $x^2$  and  $x$  and number terms.

**Simultaneous equations** are equations that are true for both variables (letters)

- To solve a simultaneous equation graphically, look at the point where both straight lines intersect (cross) and write down that coordinate.
- To solve a simultaneous equation by the elimination method, add or subtract the equations to eliminate either the  $x$  or  $y$  terms.

A term is a number, letter, or a number and a letter multiplied together i.e.  $x$ ,  $3a$ ,  $7y^2$  are all terms

An expression contains letter and/ or number terms but no equal sign.

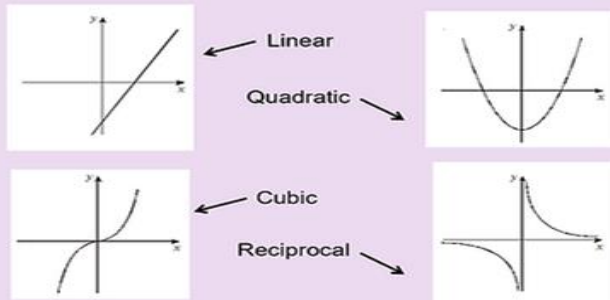
An equation has an equals sign, letter terms and numbers. You can solve it to find the value of the letter.

An identity is true for all values of letters

A formula has an equals sign and letters to represent different quantities. The letters are variables as their values can vary.

## GOOD TO KNOW...

### Types of Graphs



When rearranging formulae:

Same algebra rules apply:

- Keep equals sign in line
- Use inverse operations
- Keep the equation balanced

## HOW TO....

a Make  $a$  the subject of the formula  $v^2 = u^2 + 2as$

b Make  $x$  the subject of the formula  $y = \frac{ax+b}{c}$

$$a \quad v^2 = u^2 + 2as$$

$$b \quad y = \frac{ax+b}{c}$$

$$v^2 - u^2 = 2as \quad \text{Subtract } u^2 \text{ from both sides.}$$

$$cy = ax + b \quad \text{Multiply both sides by } c.$$

$$\frac{v^2 - u^2}{2s} = a \quad \text{Divide both sides by } 2s.$$

$$cy - b = ax \quad \text{Subtract } b \text{ from both sides.}$$

$$a = \frac{v^2 - u^2}{2s} \quad \text{Re-write in the form } a = \dots$$

$$\frac{cy - b}{a} = x \quad \text{Divide both sides by } a.$$

$$x = \frac{cy - b}{a} \quad \text{Re-write in the form } x = \dots$$

## Simultaneous Equations

By elimination method

1. Two linear equations

**Example**

Solve (i)  $2x + y = 5$

$$\begin{array}{r} x - y = 1 \\ \hline 3x = 6 \end{array} \quad \text{Eliminate } y \text{ by adding the equations}$$

$$x = 2$$

$$x - y = 1$$

$$2 - y = 1$$

Putting value back into 2<sup>nd</sup> equation

# Year 11 Term 3 Maths Knowledge Organiser [Foundation Revision]

## CORE

## GOOD TO KNOW...

## HOW TO....

### Perimeter

- Calculated by adding up the length of each of the sides.

### Circumference of circle - $\pi d$

### Area

- Square/Rectangle = length x width
- Triangle =  $\frac{1}{2} \times \text{base} \times \text{height}$
- Trapezium =  $\frac{1}{2} \times a + b \times \text{height}$
- Circle -  $\pi r^2$
- Measure in squared units, e.g.  $\text{cm}^2$

### Volume

- Cube/cuboid = length x width x height
- Prism = area of cross section (front face) x length
- Cylinder = area of circle (front face) x length
- Measure in cubic units, e.g.  $\text{cm}^3$ .

### Right Angle - $90^\circ$ Angle

Angles on a straight line =  $180^\circ$

Angles around a point =  $360^\circ$

Angles in a triangle =  $180^\circ$

Exterior angles in a polygon =  $360^\circ$

1m

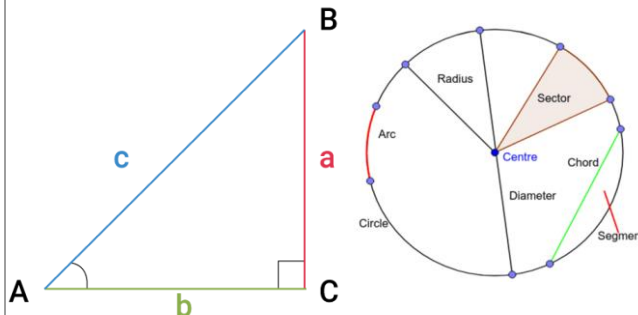
100cm



1 metre squared = 10,000 centimetre squared

### Pythagoras theorem

$$a^2 + b^2 = c^2$$



Speed

Time

Distance

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$\text{Mass} = \text{Density} \times \text{Volume}$$



**Expand**  $3(x + 4)$  Multiply what's inside by 3  
 $3x + 12$

**Factorise**  $5x - 20$  Find HCF and put in brackets  
 $5(x + 4)$

**Solve**  $3x + 4 = 40$  Use inverse operations to find value of x

$$\begin{array}{r} -4 \quad -4 \\ 3x = 36 \\ +3 \quad +3 \\ x = 12 \end{array}$$

### Percent Change

$$\text{Percent Change} = \frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100\%$$

If the result is positive, it is an increase.

If the result is negative, it is a decrease.

Increase £50 by 60%.

$$160\% \times £50 = 1.6 \times £50 = £80$$

Increase £86 by 7%.

$$107\% \times £86 = 1.07 \times £86 = £92.02$$

## CORE

**Ex1** Convert  $0.5$  to a fraction.

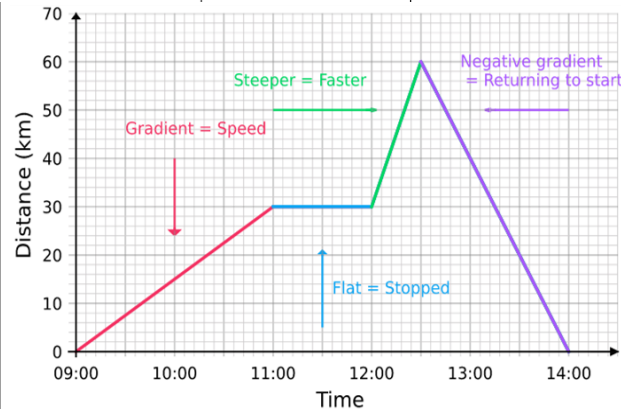
**Solution** Let  $x = 0.5$   
 $10x = 5.5$   
 $9x = 5$   
 $x = \frac{5}{9}$

**Ex2** Convert  $0.1\bar{2}$  to a fraction.

**Solution** Let  $x = 0.1\bar{2}$   
 $100x = 12.\bar{2}$   
 $99x = 12$   
 $x = \frac{12}{99} = \frac{4}{33}$

**Ex3** Convert  $0.4\bar{2}7$  to a fraction.

**Solution** Let  $x = 0.4\bar{2}7$   
 $1000x = 427.\bar{4}27$   
 $999x = 427$   
 $x = \frac{427}{999}$

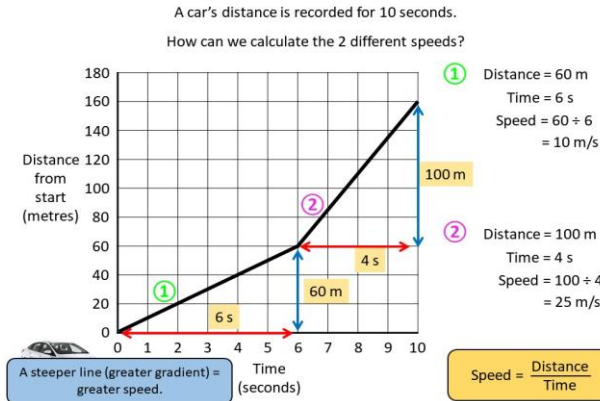


We can add vectors by adding the two x components and adding the two y components together.

$$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$$

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 + 2 \\ 4 + 7 \end{pmatrix} = \begin{pmatrix} 5 \\ 11 \end{pmatrix}$$

## GOOD TO KNOW...



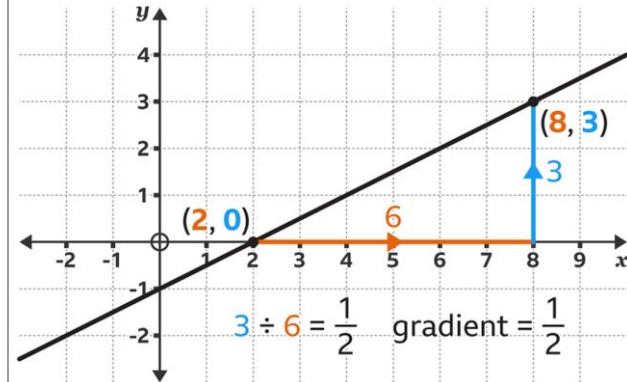
$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\text{Volume} = \frac{\text{Mass}}{\text{Density}}$$

$$\text{Mass} = \text{Density} \times \text{Volume}$$



## HOW TO....



## Percent Change

$$\text{Percent Change} = \frac{\text{New Value} - \text{Old Value}}{\text{Old Value}} \times 100\%$$

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